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THE
PRINCIPLES OF ALTERNATING CURRENTS

BY

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PREFACE.

This book has been developed from the lecture notes of the author, as given in the second year's work in Alternating Currents, in the night school at Drexel Institute. For this reason the text has been so arranged that the calculus proofs may be omitted.

After six years of teaching experience with day and evening students, the author is of the firm belief that alternating currents should as far as possible be taught graphically; and therefore vector diagrams are extensively used and always drawn so that current is used as the reference axis in series circuits, and voltage as the reference axis in parallel circuits.

Instead of a large number of manufacturers' cuts, many of which become antiquated if a book is not revised every two or three years, the author uses about one hundred lantern slides, which are constantly added to at slight expense.

The author has found from experience that the average student's knowledge of vectors is insufficient for a graphical method of treating alternating currents, and has therefore added Chapter I.

Over two hundred problems are given, nearly all of which have answers. Many principles which the student should be able to work out are given in the form of problems. The majority of the problems are short and suitable for recitation work.

Through the kindness of the Westinghouse Electric and Manufacturing Company and the General Electric Company the author is enabled to use as texts the Westinghouse bulletin on "Transmission Lines," and the General Electric bulletin on "Transformer Testing," and for this reason these two subjects have been omitted.

In the author's opinion there should be two classes of scientific books.

The first class should be called a *treatise*. This should take up the practical aspects of the subject, should contain illustrations of apparatus, and should give methods for determining proper installations, etc. It might be of any size and its chief use should be as a reference book.

The second class should be called a *text-book*. It should be primarily concerned with the principles of the subject; it should be of reasonable size and its field should be as a text for scientific students.

The present volume has been written for use with first year engineering students in Alternating Currents.

PHILADELPHIA, *Oct., 1911.*

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CHAPTER I.

VECTORS.

1. **Scalors and Vectors.**—Straight lines of definite length may be used to represent physical quantities. Some physical quantities are completely specified when their magnitude is given; *e. g.*, a sum of $10, a length of 3 feet, a flock of 7 birds, etc. Such quantities are called *scalor quantities.*

Other physical quantities have a certain direction as well as magnitude and are called *vector quantities.* The physical sense of a force of 10 pounds would not be complete unless the direction in which it acted were supplied.

In using straight lines to represent vectors, the length of the line represents the magnitude of the vector and the direction of the line represents the direction of the vector. The direction is indicated by an arrow placed on the end of the line. Thus, in Fig. 1, the line AB represents a force of 8 pounds acting horizontally toward the right.

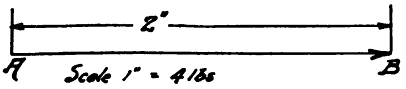


Fig 1

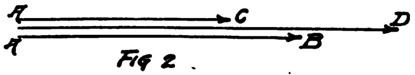


Fig 2

2. **Addition of Vectors.**—Consider the two vectors AB and AC (Fig. 2). These vectors have different magnitudes and the same direction. They are said to be in phase with each other, and their vector sum is clearly AD, which is their numerical sum (AB+AC).

Again consider the two vectors AB and AC (Fig. 3). These two vectors are the same length as those in Fig. 2, but are not in phase with each other, being 45° apart. Their vector sum is the line AD obtained by completing the parallelogram. Clearly AD of Fig. 3 is less than the AD of Fig. 2.

The method here given is evident when we consider that the projection of the diagonal of a parallelogram on any line is equal to the sum of the projections of the two adjacent sides of the parallelogram on the same line.

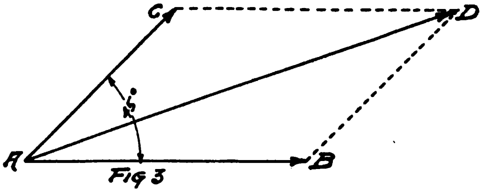


Fig 3

In Fig. 4 the same two lines AB and AC of Figs. 2 and 3 are shown 120° apart. Their vector sum is again AD as obtained in Fig. 3, but it is less in magnitude than in Fig. 3.

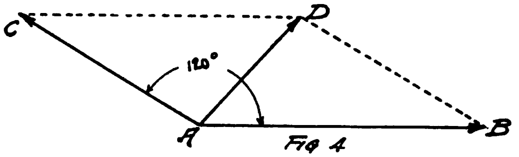


Fig 4

In Fig. 5 AB and AC are 180° apart and their vector sum is AD less than in any of the previous cases and equal to their numerical difference (AB−AC), and in the direction of the larger one (AB).

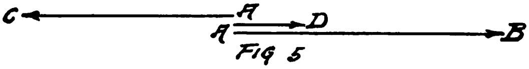


Fig 5

From the figures it is shown that the vector sum of two quantities is a maximum when the two quantities are in phase (Fig. 2) and the vector sum of two quantities is a minimum when they are 180° apart (Fig. 5).

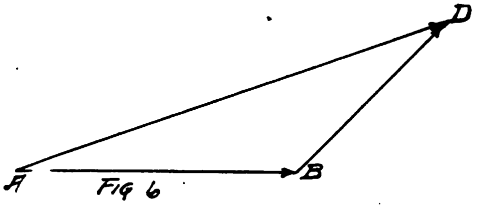
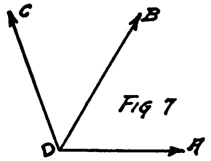
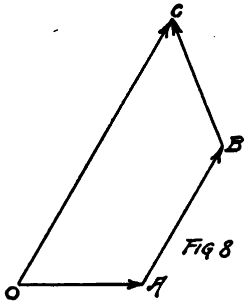


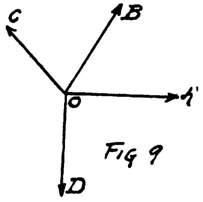
Fig 6

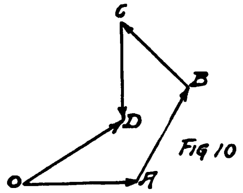
In Fig. 3 the vector sum AD could be obtained more easily as shown in Fig. 6. This construction removes the necessity for constructing the two sides AC and CD of the parallelogram shown in Fig. 3. Similarly for Fig. 4 or for any case.





Addition of More Than Two Vectors.—In Fig. 7 consider the three vectors OA, OB, and OC with the magnitudes and directions as shown. Their vector sum (Fig. 8) is OC, and is obtained by drawing OA equal and parallel to OA (Fig. 7) and from the end of OA drawing AB equal and parallel to OB (Fig. 7) and from the end of B drawing BC equal and parallel to OC (Fig. 7).

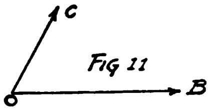


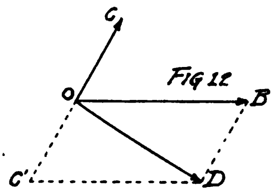


In Fig. 10, OD is the vector sum of the four vectors shown in Fig. 9.

(Note.—This subject is more fully developed in chapter II.)

3. **The Subtraction of Two Vectors.**—Consider the two vectors OB and OC (Fig. 11). It is required to find their vector difference (OB − OC). Reverse the vector with the minus sign before it,





OC (Fig. 12), giving OC′, and complete the parallelogram, just as when adding two vectors, giving as a result OD = (OB − OC).

Again consider the two vectors OB and OC (Fig. 11). It is required to find their vector difference (OC − OB). Reverse the vector with the minus sign before it, OB (Fig. 13), giving OB′, and complete the parallelogram, giving as a result OD = (OC − OB).

The OD of Fig. 13 is equal to the OD of Fig. 12, but 180° away from it in direction. This is always the case.

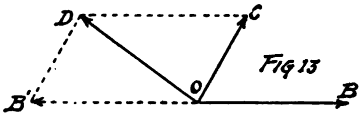


Fig 13

The vector difference of two quantities is a minimum when the two quantities are in phase, and is a maximum when they are 180° apart.

4. **Addition and Subtraction of More Than Two Vectors.**—In Fig. 14 consider the vectors OA, OB, OC, and OD with the magnitudes and directions as shown. Find OA − OB + OC − OD. In Fig. 15, from the point O draw OA equal to, parallel to, and in the same direction as OA (Fig. 14); from A draw AB′ equal to, parallel to, and in the opposite direction to OB (Fig. 14); from B′ draw B′C equal to, parallel to, and in the same direction as OC (Fig. 14); from C draw CD′ equal to, parallel to, and in the opposite direction to OD (Fig. 14). The line OD′ (Fig. 15) = OA − OB + OC − OD.

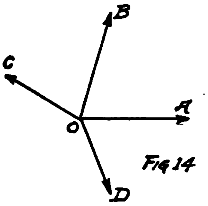


Fig 14

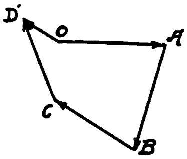


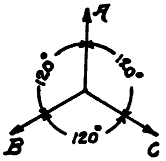
Fig 15

Follow this general rule:—

Reverse the direction of all vectors having the minus sign before them.

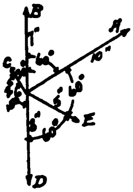
PROBLEMS.

1. Add two vectors each 10 inches long and 120° apart. *Ans.:*—10 inches.
2. Subtract two vectors each 10 inches long and 60° apart. *Ans.:*—10 inches.
3. Add two vectors each 10 inches long and 60° apart. *Ans.:*—17.32 inches.
4. Subtract two vectors each 10 inches long and 120° apart. *Ans.:*—17.32 inches.
5. Consider the three vectors shown in the figure each 10 inches long, find:—



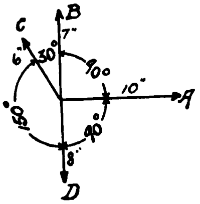
(a)	OA+OB+OC.	*Ans.:*—0.
(b)	OA+OB−OC.	*Ans.:*—20 inches.
(c)	OA−OB+OC.	*Ans.:*—20 inches.
(d)	OA−OB−OC.	*Ans.:*—20 inches.
(e)	−OA−OB+OC.	*Ans.:*—20 inches.
(f)	−OA−OB−OC.	Compare part (f) with part (a).

6. In the figure shown find:—



(a) OA+OB+OC+OD+OE.
(b) OA−OB+OC−OD+OE.
(c) OA+OB−OC−OD−OE.
(d) −OA−OB−OC−OD−OE.
Compare part (d) with part (a).

7. Add the vectors A, B, C, D in the order given. Then add them in the order A, C, D, B. Compare the two results. What do you conclude?



8. One vector 8 inches long is 90° from another 6 inches long; find their sum by two methods. *Ans.:*—10 inches.
9. What happens to the sum of two vectors as you increase the angle between them?
10. What happens to the difference of two vectors as you increase the angle between them?
11. Draw two unequal vectors, OA and OB, 45° apart.

Find (a) OA−OB } Compare part a and part b as to magnitude.
(b) OB−OA } Compare part a and part b as to direction.
(c) OA+OB } Compare part c and part d as to magnitude.
(d) −OA−OB } Compare part c and part d as to direction.

CHAPTER II.

ALTERNATING CURRENT CONCEPTIONS.

5. **An Alternating Current.**—Consider a hydraulic pump connected to the pipe line shown (Fig. 16). When the plunger is moving up, water is flowing as indicated by the full-line arrows past the points a and b. When the plunger moves down, the water flows as indicated by the dotted-line arrows past the points a and b.

Consider an alternating current generator (g) connected to the circuit shown (Fig. 17). At a given instant electricity will be flowing from the generator as indicated by the full-line arrows, and the positive terminal of the generator is the upper one in the figure.

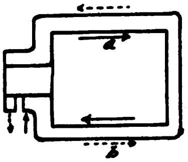


Fig 16

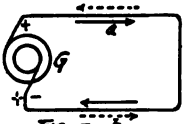


Fig 17

An instant later the positive terminal of the generator is the lower one and electricity will be flowing through the circuit as indicated by the dotted-line arrows.

Such a current of water is called an alternating current of water. Such a current of electricity is called an alternating current of electricity.

6. **The Production of an Alternating Current.**—Let us consider

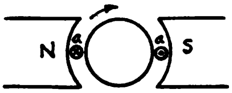


Fig 18

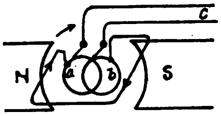


Fig 19

a two-pole D.C. generator (Fig. 18), rotating clockwise. When the conductor (a) on the armature is in front of an N-pole, the current is flowing away from reader, and one-half a revolution further, or when it is under a S-pole, the current is flowing toward the reader. Therefore, we see that all D.C. machines have alternating current generated in their armatures, and it is rectified into direct current by means of a commutator.

If instead of connecting the armature winding to a commutator it is connected to two copper rings (a) and (b) (Fig. 19), the current delivered to the external circuit (c) is alternating current. Such a machine is an A.C. generator.

7. **Wave Shapes.**—Consider a multipolar A.C. generator NSN, etc. (Fig. 20). It is required to plot the e.m.f. wave of this generator. For simplicity it is assumed that there is a uniform distribution of flux over each N and S-pole. Let the surface of the armature be *xx*. The conductor (t) on the armature moves from left to right as indicated by the arrow and is not cutting any flux and therefore has no e.m.f. induced in it until it reaches the edge of the pole (a). It then begins to cut flux and has an e.m.f. of say 10 volts induced in it away from the reader. Since the distribution of flux is uniform across the pole face, the voltage stays at 10 volts until point (b) is reached when it falls to zero. From (b) to (c) no flux is cut, therefore no e.m.f. is induced. From (c) to (d) the same amount of flux is cut as the conductor passes under an S-pole, therefore an e.m.f. of 10 volts will be induced but toward the reader. The rest of the curve is self-explanatory.

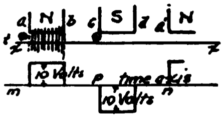


Fig 20

The curve shown in Fig. 21 is similar to Fig. 20, except that there is fringing of the flux at each pole tip.

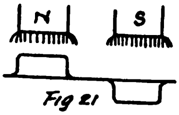


Fig 21

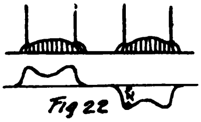


Fig 22

Fig. 22 shows how the variation in the shape of the pole shoe can effect the wave shape; such a pole is never used.

8. **Definitions.**—A *cycle* is one complete set of + and − values;
the distance mn (Fig. 20).

An *alternation* is one-half of a cycle;
the distance mp (Fig. 20).

A 60-cycle generator is one which completes 60 cycles every second or 120 alternations per second.

Frequency is the number of cycles in one second. The commonest frequencies in practice are 15 for power only, 25 generally power, 60 power and lighting, 133 generally lighting.

Period is the time in seconds for 1 cycle.

In a 60-cycle circuit the period is one-sixtieth of a second.

Amplitude is the maximum ordinate of a wave.

The amplitude of the wave (Fig. 22) is the distance x.

9. **Formula.**—To complete one cycle, each conductor on the armature must pass one N and one S-pole.

Let p = number of poles on an A.C. generator.
Let n = speed of the armature in r.p.s.
Let f = frequency.

Then $\frac{p}{2}$ = cycles completed in each revolution of the armature.

And $\frac{p}{2} \times n$ = cycles completed in one second = frequency,

or $$\frac{pn}{2} = f \qquad (1)$$

Example:—

A four-pole A.C. generator runs at 1800 r.p.m.:

In this case $p=4$, $n=\frac{1800}{60}=30$ and $f=\frac{4\times30}{2}=60$ cycles per second.

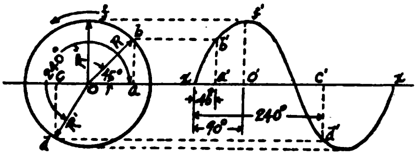


Fig 23

10. **A Sine Wave.**—Consider the radius R (Fig. 23) to be rotating about the center O, with a uniform velocity in a counter clockwise direction as shown by the arrow. Lay off the straight line xx equal to 360° drawn to some convenient scale. This line is to represent the x-axis of the wave. Consider the radius R to be 45° from the horizontal as shown. From the point x measure to the right a distance of 45° to scale and at this point (a′) erect a perpendicular (a′b′) equal to (ab) the vertical projection of R. This is one point on the curve. Again assume that at a given instant, the revolving radius has reached the position R′ 240° from where it passes through the horizontal on the right. To obtain the corresponding point on the sine wave, from the point x measure to the right a distance of 240° to scale and at this point

erect a perpendicular (c′d′) equal to (cd) the vertical projection of R′. In this way the sine wave may be obtained from the rotating radius.

From the construction $a'b' = ab = R \sin 45°$ because $\sin 45° = \frac{ab}{R}$

also $c'd' = cd = R \sin 240°$ etc.

It is noticed, further, that the maximum value any ordinate can have is when R is 90° from the horizontal and from the construction $o'f' = of = R \sin 90°$ but $\sin 90° = 1$, $of = R \times 1 = R$.

Because each instantaneous ordinate equals the maximum ordinate times the sine of the angle the maximum ordinate is from the horizontal (Fig. 23), it is called a sine wave. The rotating diagram from which the sine wave is constructed is called a vector or clock diagram. It is assumed throughout this syllabus that all vector diagrams rotate counter clockwise. If the clock diagram (Fig. 23) represents a 60-cycle generator, then the vector (R) revolves about the point 0, 60 times every second. This sine wave can represent a wave of e.m.f. by changing R to E. Then any instantaneous e.m.f. $e = E \sin \alpha$—where α is the angle E happens to be from the horizontal (Fig. 23) and e is the instantaneous e.m.f. at that instant. Similarly, this sine wave can represent a wave of current by changing R to I. Then any instantaneous current $i = I \sin \alpha$.

While A.C. generator waves are not sine waves, their effect is so nearly the effect of a sine wave that sine waves are nearly always assumed because they are so much easier to handle in making calculations.

From Fig. 23 it is noticed that 1 cycle = 360° so that when a conductor on the armature has passed one N and one S-pole it has completed 360° electrical degrees. On a two-pole generator 360° electrical degrees = 360 mechanical degrees; on a four-pole generator 360° electrical degrees = 180 mechanical degrees, etc.

	$e = E \sin \alpha$		
at point a (Fig. 24)	$e = 100 \sin 0 =$	0	volts
at point b	$e = 100 \sin 45° =$	70.7	volts
at point c	$e = 100 \sin 180° =$	0	volts
at point d	$e = 100 \sin 270° =$	−100	volts
at point e	$e = 100 \sin 360° =$	0	volts

From Fig. 24 it is seen that the e.m.f. wave is steeper when it crosses the axis than when it is at its + or — maximum; in other words, the e.m.f. is changing more rapidly at the points 0° and 180° than at the points 90° and 270°. (See art. 20.)

11. **Addition of Sine Waves.**—Let the two vectors op and op′ represent two e.m.f.'s, both passing through the horizontal at the same instant; then by the method of Fig. 23 op will produce the sine wave E and op′ will produce the sine wave E′ both of which pass through zero at the same time. When such is the case the two e.m.f.'s are said to be in phase with each other, and since op and op′ rotate counter clockwise at the same speed the two e.m.f.'s have the same frequency.

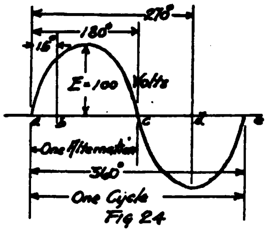


Fig 24

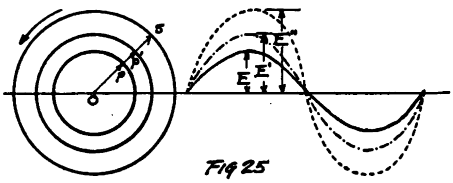


Fig 25

Let the line os = op + op′. Therefore, os is constant in length and rotates counter clockwise with the same velocity as op or op′, and is in phase with op and op′ and produces the dotted sine wave E″ which is in phase with E and E′.

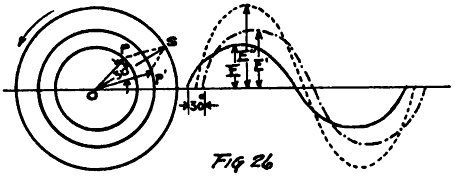


Fig 26

Let the two vectors op and op′ represent two e.m.f.'s and suppose that op passes through the horizontal 30° ahead of op′ but that both rotate counter clockwise with the same velocity. As before, these two vectors op and op′ will produce the two sine waves E and E′. Completing the parallelogram we get os equal the vector sum of op and op′. Now, os will also rotate counter clockwise and is constant in length and will therefore also produce a true sine wave E″. The two waves E and E′ have the same frequency, but are not in phase with each other as in Fig. 25. Since op is ahead of op′ 30°, so E is said to be ahead of E′ 30°. Further, it will be noticed that os (Fig. 25) is larger than os (Fig. 26); the resultant sine wave is a maximum when the waves of which it is composed are in phase. From Fig. 26 it is seen that the resultant wave E″ is slightly behind E and slightly ahead of E′.

The wave representing the sum of two or more sine waves may be obtained by adding together the ordinates or instantaneous values of the original waves.

When two or more waves pass through their zero values together, they are said to be in phase with each other.

When two or more waves do not pass through their zero values together, they are said to be out of phase. In Fig. 26 E is said to lead E′ by 30°.

In this case 30° is said to be the angle of lead of E over E′.

Again in Fig. 26 E′ is said to lag behind E by 30°.

In this case 30° is said to be the angle of lag of E′ behind E.

If two 110 volt D.C. dynamos are connected in series, the resultant e.m.f. is 220 volts. In direct currents voltages and currents are always in phase with each other. In alternating currents voltages and currents may be in phase (Fig. 25) but generally are not in phase (Fig. 26); this is one of the facts that has given A.C. its unenviable reputation.

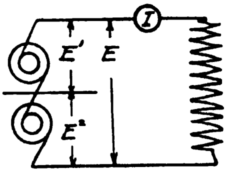


Fig 27

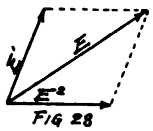


Fig 28

12. Examples.—Example No. 1; the addition of e.m.f.'s:—

Consider the two A.C. generators E_1 and E_2 (Fig. 27) connected in series. Both generators have the same frequency but their e.m.f.'s are not in phase as shown (Fig. 28). The resultant voltage E is the vector sum of E_1 and E_2.

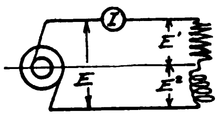


Fig 29

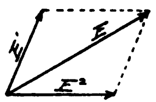


Fig 30

Example No. 2; the addition of e.m.f.'s:—

Consider the two receiving circuits E_1 and E_2 (Fig. 29) connected in series. The total voltage of the generator (E) is clearly the vector sum of E_1 and E_2 (Fig. 30).

Both of these examples are series circuits and have but one current (I).

Example No. 3; the addition of currents:—

Consider the two A.C. generators I_1 and I_2 (Fig. 31) connected in parallel. Both generators have the same frequency but their currents are not in phase as shown (Fig. 32). The resultant I is the vector sum of I_1 and I_2.

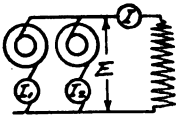


Fig 31

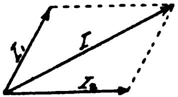


Fig 32

Example No. 4; the addition of currents:—

Consider two receiving circuits I_1 and I_2 (Fig. 33) connected in parallel. The total current (I) supplied by the generator is clearly the vector sum of I_1 and I_2 as shown (Fig. 34).

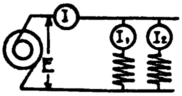


Fig 33

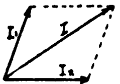


Fig 34

Both of these examples 3 and 4 are parallel circuits and have but one voltage (E).

13. **A Trigonometric Method of Getting the Sum of Two or More Vectors not in Phase with Each Other.**—Consider the three receiving circuits x, y, and z (Fig. 35) having the currents I_1, I_2, and I_3. It is required to find the total current (I).

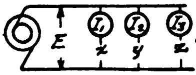


Fig 35

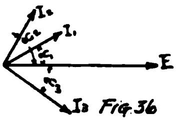


Fig 36

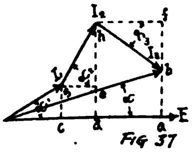


Fig 37

E is common to all three circuits, and is therefore drawn along the horizontal and used as the reference axis (Fig. 36). Suppose I_1 leads E by α_1 degrees, I_2 leads E by α_2 degrees, and I_3 lags behind E by α_2 degrees.

From the point O (Fig. 37) draw OI_1 equal and parallel to I_1 (Fig. 36); from the end of I_1 draw $I_1 I_2$ equal and parallel to I_2 (Fig. 36) and from the end of I_2 draw $I_2 I_3$ equal and parallel to I_3 (Fig. 36); then the line joining O and I_3 is I the vector sum of I_1, I_2, and I_3.

From the geometry of Fig. 37, $I=\sqrt{(oa)^2+(ab)^2}$ (2)
drop perpendiculars from the ends of I_1, I_2, and I_3 on E (see Fig. 37),

$$\begin{aligned} \text{then } oa &= oc+cd+da \\ &= I_1 \cos \alpha_1 + I_2 \cos \alpha_2 + I_3 \cos \alpha_3 \end{aligned}$$

drop lines parallel to E from the ends of I_1, I_2, and I_3 (see Fig. 37),

$$\begin{aligned} \text{then } ab &= cg+eh-bf \\ &= I_1 \sin \alpha_1 + I_2 \sin \alpha_2 - I_3 \sin \alpha_3 \end{aligned}$$

Substituting in equation (2) we have

$$I=\sqrt{(I_1 \cos \alpha_1 + I_2 \cos \alpha_2 + I_3 \cos \alpha_3)^2 + (I_1 \sin \alpha_1 + I_2 \sin \alpha_2 - I_3 \sin \alpha_3)^2} \quad (3)$$

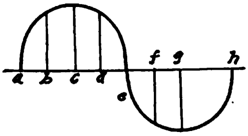
From trigonometry

$$\cos \alpha = \frac{I_1 \cos \alpha_1 + I_2 \cos \alpha_2 + I_3 \cos \alpha_3}{I} \quad (4)$$

This current and this value of α should check with those obtained graphically.

PROBLEMS.

12. A 10-pole generator completes how many (a) alternations, (b) cycles in one revolution of its armature? *Ans.:*—(a) 10, (b) 5.
13. What must be the speed of a 60-cycle 4-pole generator?
14. What is the frequency of a 10-pole generator running 1590 r.p.m.?
15. How many poles has a 25-cycle generator running 75 r.p.m.?
16. A 15-cycle generator runs at 450 r.p.m. How many poles does it have? What must be the speed of the same generator so that it can operate and supply 60-cycle mains?
17. What is the effect on the frequency of a given machine, operating at a fixed speed, if the number of poles is doubled? What is the effect on the frequency of a given machine, if its speed is doubled?
18. A rheostat of 12 ohms resistance is inserted in a circuit in which a steady current of 20 amperes is flowing; it is short circuited three-fourths of the time. What is the equivalent constant resistance of the rheostat? *Ans.:*—3 ohms.
19. The rheostat of problem 18 is short circuited 50 times per second, and each time for one-two-hundredth of a second. What will be the equivalent constant resistance of the rheostat? *Ans.:*—9 ohms.
20. Plot a curve of the instantaneous resistance of prob. 19; resistance in ohms as ordinates; time in seconds as abscissæ.
21. An e.m.f. of 200 volts acts on a circuit containing a resistance of 20 ohms. The e.m.f. is reversed every .01 of a second. Plot the e.m.f. curve and the current curve.
22. The maximum value of the sine wave of e.m.f. shown is 100 volts. Find its instantaneous value at the points a, b, c, d, e, f, g, and h. *Ans.:*—(a) 0, (b) +70.7, (c) + 100, (d) −, (e) −, (f) −70.7, (g) −, (h) −.



23. The maximum value of a 60-cycle sine wave of current is 100 amperes. What will be its instantaneous value (a) 1 second, (b) 2 seconds, (c) $\frac{1}{60}$ second, (d) $\frac{1}{120}$ second, (e) ½ second, (f) ¼ second, after passing the point a. (See figure in problem 22.)

24. In problem 23 how many complete cycles has the wave of current passed through in each case? *Ans.*:—(a) 60, (b) –, (c) –, (d) ½ cycle, (e), –, (f) –.

25. The maximum value of a 60-cycle sine wave of current is 10 amperes. Find its instantaneous value (a) $\frac{1}{1000}$ second, (b) $\frac{3}{50}$ second, (c) $2\frac{1}{5}$ seconds, after passing the point (a). (See figure in problem 22.) *Ans.*:—(a) +3.68 (b) –5.88 (c) o.

26. In problem 25 what will be the instantaneous values in each case after passing through the point c? (See figure in problem 22.) *Ans.*:—(a) +9.3, (b) –8.09, (c) +10.

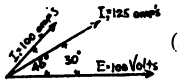
27. What is the maximum value of a sine wave of current, when the instantaneous value of the current is 5 amperes, 30° after the beginning of a cycle? *Ans.*:—10 amperes.

28. What is the frequency of an e.m.f. wave which repeats itself every .01 second?

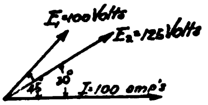
(Do the rest of the problems graphically and by trigonometry and check results. Always make a vector diagram.)

29. Two motors are connected in parallel to the same circuit. The first takes 100 amperes and the second takes 100 amperes. These two currents are 90° apart. What is the total current?

30. In the circuit shown (Fig. a) find (a) the total current (I). (b) Does this represent a series or a parallel circuit? (c) What is the voltage across each circuit? *Ans.*:—(a) 223 amperes; (b) –, (c) –.

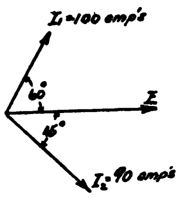


(a)



(b)

31. In the circuit shown (Fig. b) find (a) the total voltage E. (b) Is it ahead or behind I and by how many degrees?



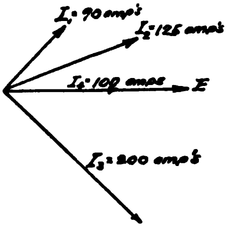
32. In the parallel circuit shown find the total current I. Is it ahead or behind E and by how many degrees? *Ans.*:—I = 116 amperes and is ahead of E 11° 15″.

33. Show, using a vector diagram, how to add two sine waves 45° apart.

34. Make a clock diagram of the e.m.f.'s and currents in three receiving circuits connected in parallel to a generator.

35. Make a clock diagram of the e.m.f.'s and currents in three receiving circuits connected in series to a generator.

36. Find the total current in the circuit shown in the figure. How many de grees is it ahead or behind E? *Ans.*:— 394.66 amperes and is slightly behind E.



37. Four 25 cycle generators are connected in series, generating respectively 200, 160, 180, and 100 volts. What will be the value of the resultant voltage if the phase difference between successive components is 45°? (Use any one of the four as the x-axis.) *Ans.*:— 438 volts.

38. Derive a general expression for obtaining the vector sum of any number of vectors not in phase with each other.

CHAPTER III.

AVERAGE; ROOT MEAN SQUARE; POWER.

14. **Average e.m.f. or Current.**—By the average e.m.f. or current is meant the average of all the instantaneous values in one half-cycle. Clearly the average of all the instantaneous values for any wave for one whole cycle is zero, as there are just as many positive values as negative ones. Now, the average height of any figure is equal to the $\frac{\text{area}}{\text{base}}$, since area = base × average height.

Examples.—

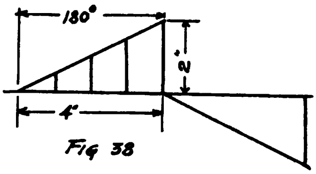


Fig 38

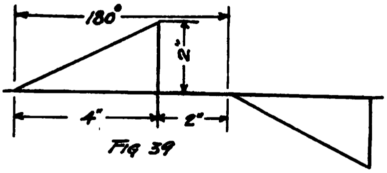


Fig 39

$$\text{In Fig. 38 Av. height} = \frac{\text{area}}{\text{base}} = \frac{4 \text{ sq. in.}}{4 \text{ in.}} = 1 \text{ in.}$$

$$\text{In Fig. 39 Av. height} = \frac{\text{area}}{\text{base}} = \frac{4 \text{ sq. in.}}{6 \text{ in.}} = \frac{2}{3} \text{ in.}$$

In the case of a sine wave, again, the average height is equal to the $\frac{\text{area}}{\text{base}}$, but without the use of the calculus it is quite difficult to find the area of one lobe (one half-cycle) of a sine wave. A very accurate and easy way is to divide the one half-cycle (180°) into ten or more equal parts (the more parts, the more accurate the result); add these instantaneous heights by getting the sines of the angles from 0° to 180° from a trigonometric table and divide this sum by the number of parts taken, thus in Fig. 40:—

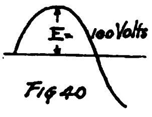


Fig 40

$e_1 = E \sin 10° = 100 \times .174 =$	17.4	$E \sin 70° =$	930.
$e_2 = E \sin 20° = 100 \times .342 =$	34.2	$E \sin 60° =$	86.6
$E \sin 30° = 100 \times .5 =$	50.0	$E \sin 50° =$	76.6
$E \sin 40° =$	= 64.3	$E \sin 40° =$	64.3
$E \sin 50° =$	= 76.6	$E \sin 30° =$	50.0
$E \sin 60° =$	= 86.6	$E \sin 20° =$	34.2
$E \sin 70° =$	= 93.0	$E \sin 10° =$	17.4
$E \sin 80° =$	98.5	$E \sin 0° =$	0
$E \sin 90° =$	100.0		
$E \sin 80° =$	98.5		1141.2

Dividing this number by the number of parts (18), we obtain $\frac{1141.2}{18}=63.4$ volts. Worked out accurately by the method of the calculus, the result is 63.6 volts, which is equal to $\frac{2}{\pi}\times 100$.

Therefore the average height of a sine wave equals the maximum height × .636 or

$$e_{\text{(average)}}=.636\ E=\frac{2}{\pi}\ E \qquad (5)$$

$$\text{and } i_{\text{(average)}}=.636\ I=\frac{2}{\pi}\ I.$$

15. Root Mean Square e.m.f. or Current.—In direct currents the heating effect depends on I^2R or varies as I^2. If it is desired to find what A.C. current will produce the same heating effect as a D.C. current, it must evidently be some sine wave of current the square of each ordinate through a cycle producing the same heating effect as the square of the D.C. current.

Therefore, the following operation on the sine wave is necessary:

First: Square each ordinate of the sine wave.

Second: Add the squares together and divide by their number. This gives the average or mean square.

Third: Extract the square root. This gives the square root of the average or mean square, or called more briefly root mean square (r.m.s.), though many authors use the word effective instead of r.m.s.

Examples.—

In Fig. 38 divide the one half-cycle into four equal parts.

First: $\left(\frac{1}{2}\right)^2+(1)^2+\left(\frac{3}{2}\right)^2+(2)^2=7\frac{1}{2}.$

Second: $\frac{7\frac{1}{2}}{4} \qquad =1.875$

Third: $\sqrt{1.875} \qquad =1.37$ ins. Ans.

In Fig. 39 divide the one half-cycle into six equal parts.

First: $\left(\frac{1}{2}\right)^2+(1)^2+\left(\frac{3}{2}\right)^2+(2)^2+0^2+0^2=7\frac{1}{2}.$

Second: $\frac{7\frac{1}{2}}{6} \qquad =1.25.$

Third: $\sqrt{1.25} \qquad =1.117$ ins. Ans.

Performing the same three operations for the example of the sine wave (Fig. 40) and dividing by 18, we get—

First: $(17.4)^2+(34.2)^2+(50)^2+\text{etc.}=89624.4.$

Second: $\dfrac{89624.4}{18} = 4980.$

Third: $\sqrt{4980} = 70.6$ volts. Ans.

As in the case of the average e.m.f., the more parts the wave is divided into the more accurate will be the result. Worked out accurately by the method of the calculus, the result is 70.7 volts, which is equal to $\dfrac{1}{\sqrt{2}}\times 100$, or—

$$e_{(\text{r.m.s.})}=70.7\ E=\frac{E}{\sqrt{2}}. \qquad (6)$$

$$\text{and } i_{(\text{r.m.s.})}=70.7\ I=\frac{I}{\sqrt{2}}.$$

$$\begin{matrix}\text{Where} \\ \text{and}\end{matrix}\left\{\begin{matrix}E=\text{maximum value of e.m.f.} \\ I=\text{maximum value of current.}\end{matrix}\right.$$

16. **Form Factor.**—The form factor in any A.C. circuit is equal to the r.m.s. height divided by the average height. In the case of a sine wave the form factor $=\dfrac{.707\ E}{.636\ E}=1.11.$

17. **Calculus Method for Obtaining Av. Height and r.m.s. Height.**—1. To find the average height of a straight line curve:

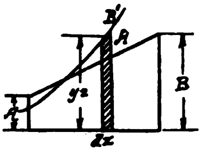


Fig 41

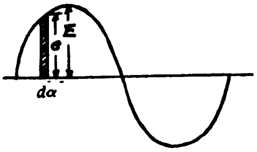


Fig 42

Obtain the area by integrating a small differential area between the proper limits.

2. To find the r.m.s. height of a straight line curve. The equation of the curve A (Fig. 41) is $x=ay$.

Curve B is obtained by squaring the ordinates of curve A.

The small differential area (shaded) $=y^2\,dx$

$$\text{but } dx=a\,dy$$

$$\text{Area under curve B}=a\int_A^B y^2\,dy$$

$$\text{Area}=a\,\frac{1}{3}\,(B^3-A^3)$$

$$\text{Av. square height}=\frac{a\,\frac{1}{3}(B^3-A^3)}{\text{base}}=\frac{a\,\frac{1}{3}\,(B^3-A^3)}{a\,(B-A)}$$

$$\text{r.m.s. height}=\sqrt{\frac{1}{3}\,\frac{B^3-A^3}{B-A}}$$

3. To find the average height of a sine curve of e.m.f. The equation of the curve A (Fig. 42) is $e = E \sin \alpha$.

$$\text{The small differential area (shaded)} = e d\alpha$$
$$\text{but } e = E \sin \alpha$$
$$\therefore \text{area} = E \sin \alpha \, d\alpha$$

Integrating between the limits of π and o.

$$\text{Area} = \int_o^{\pi} \sin \alpha \, d\alpha$$
$$= E \, (-\cos \alpha)_o^{\pi}$$
$$= E - (-1 - [+1]) = 2 \, E$$
$$\text{Av. height } e_{(av.)} = \frac{\text{area}}{\text{base}} = \frac{2 \, E}{\pi} \text{ and } i_{(av.)} = \frac{2}{\pi} \, I$$

4. To find the r.m.s. height of a sine wave of e.m.f. Curve B (Fig. 43) is obtained by squaring the ordinates of curve A.

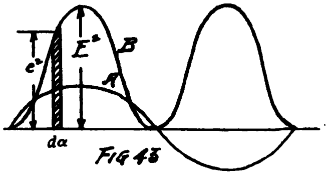


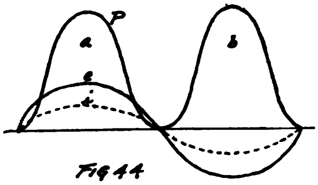
FIG 43

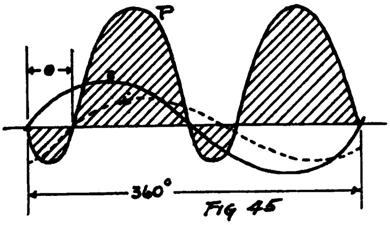
$$\text{The small differential area (shaded)} = e^2 d\alpha$$
$$\text{but since } e = E \sin \alpha, \; e^2 = E^2 \sin^2 \alpha$$
$$\text{and area} = E^2 \sin^2 \alpha \, d\alpha$$
$$\sin^2 \alpha = \frac{1 - \cos 2 \, \alpha}{2} = \frac{1}{2} \, (1 - \cos 2 \, \alpha)$$
$$\therefore \text{area} = \frac{E^2}{2} \, (1 - \cos 2 \, \alpha) \, d\alpha$$

Integrating between the limits π and o.

$$\text{area} = \frac{E^2}{2} \int_o^{\pi} d\alpha - \int_o^{\pi} \cos 2\alpha \, d\alpha$$
$$= \frac{E^2}{2} \left[(\alpha)_o^{\pi} - \frac{1}{2} \, (\sin 2 \, \alpha)_o^{\pi} \right]$$
$$= \frac{E^2}{2} \quad \pi - o$$
$$\text{or average square height} = \frac{E^2 \pi}{2 \text{ base}} = \frac{E^2 \pi}{2\pi} = \frac{E^2}{2}$$
$$\text{or r.m.s. height, } e_{(r.m.s.)} = \sqrt{\frac{E^2}{2}} = \frac{E}{\sqrt{2}} \text{ and } i^{(r.m.s.)} = \frac{I}{\sqrt{2}}$$

18. Power.—In direct currents power P is equal to e×i. In alternating currents the power constantly varies and at each instant is equal to the instantaneous e.m.f. at that instant times the instantaneous current at the same instant. Thus in Fig. 44 the two sine waves represent a wave of current and a wave of e.m.f., in phase with each other. The ordinates of the high curve (P), which is the power curve, are obtained by multiplying the ordinates of the other two curves together. Since two negatives make a positive, the P curve in both half-cycles is positive. This curve shows that the generator gives out two pulses of power, a and b, every cycle.





In Fig. 45 the e.m.f. wave is ahead of the current wave by θ degrees, and the power curve is obtained just as in Fig. 44. Twice during each cycle the generator is delivering power and acting as a generator, shown by the two larger shaded areas; and twice during each cycle it is receiving power and acting as a motor, shown by the two smaller shaded areas.

In Fig. 46 the wave of e.m.f. is 90° ahead of the current wave, and twice each cycle the generator is delivering power and twice each cycle the generator is receiving power, and since the negative lobes of the power curve are equal to the positive lobes, the resultant power is zero and the generator takes as little power from the prime mover as if the main switch were open while the ammeter may be reading full load current and the voltmeter normal voltage.

Further analysis shows that if the angle between e and i exceeds 90°, the power becomes negative, and the generator becomes a motor and receives power from the line. These last two conditions never exist in actual practice.

19. To Derive an Expression for Power.—Assume a sine wave of e.m.f. and a sine wave of current in which the current wave lags behind the e.m.f. wave θ degrees (Fig. 47).

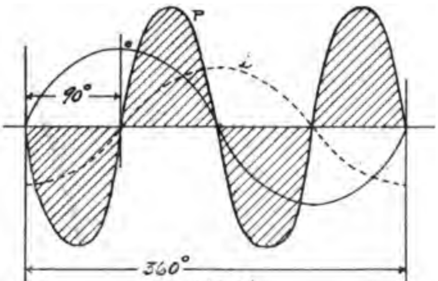


Fig 46

$$\text{From Fig. 47 the instantaneous e.m.f.} = e = E \sin(\alpha+\theta) \qquad (7)$$
$$\text{and instantaneous current} = i = I \sin\alpha \qquad (8)$$

$$\text{Now instantaneous power } P = e \times i = E \sin(\alpha+\theta) \times I \sin\alpha$$
$$\text{but } \sin(\alpha+\theta) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$$

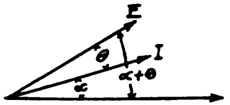


Fig 47

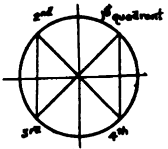


Fig 48

$$P_{(\text{inst})} = E(\sin\alpha\cos\theta + \cos\alpha\sin\theta) \times I \sin\alpha$$
$$= E\,I(\sin\alpha^2\cos\theta + \cos\alpha\sin\alpha\sin\theta)$$
$$P_{(\text{average})} = E\,I(\cos\theta \text{ av. } \sin^2\alpha + \sin\theta \text{ av. } \cos\alpha\sin\alpha) \qquad (a)$$
$$\text{Since E, I, and } \cos\theta \text{ are constants}$$

In 1st quad av. $\sin\alpha = +\frac{1}{\sqrt{2}}$ (see Fig. 48), since $\begin{cases} \sin 0° = 0 \\ \sin 45° = +\frac{1}{\sqrt{2}} \\ \sin 90° = 1 \end{cases}$

In 1st quad av. $\sin^2\alpha = \frac{1}{2}$

In 2d quad av. $\sin\alpha = +\frac{1}{\sqrt{2}}$ (see Fig. 48)

In 2d quad av. $\sin^2\alpha = \frac{1}{2}$

In 3d quad av. $\sin\alpha = -\frac{1}{\sqrt{2}}$ (see Fig. 48)

In 3d quad av. $\sin^2\alpha = +\frac{1}{2}$

In 4th quad av. $\sin\alpha = -\frac{1}{\sqrt{2}}$ (see Fig. 48)

In 4th quad av. $\sin^2\alpha = +\frac{1}{2}$

Av. $\sin^2\alpha$ through 1 cycle or 360° or 4 quadrants $= \frac{1}{2}$

by a similar analysis

Av. $\cos\alpha \sin\alpha = 0$ (Make this analysis)

$$\therefore \quad P_{(\text{average})} = E\,I\left(\frac{1}{2}\cos\theta\right) = \frac{1}{2}\,E\,I\cos\theta \qquad (9)$$

$$\text{but } E = e_{(\text{r.m.s.})} \times \sqrt{2}$$

$$\text{and } I = i_{(\text{r.m.s.})} \times \sqrt{2}$$

$$\therefore \quad P_{(\text{average})} = \frac{1}{2} \times e\sqrt{2} \times i\sqrt{2}\cos\theta$$

$$= e_{(\text{r.m.s.})}\, i_{(\text{r.m.s.})} \cos\theta \qquad (10)$$

This result may be more readily obtained by integrating equation (a) between the limits of 2π (or π) and 0.

Cos θ is called the power factor and is the cosine of the angle between the e.m.f. and current in any circuit.

When the e.m.f. is ahead of the current, the power factor is considered positive; when the e.m.f. is behind the current, the p.f. is considered negative. In most practical cases the p.f. is positive and with a purely lamp load it is practically unity. (e and i are in phase with each other.)

20. Rate of Change.—Consider a sine wave of e.m.f.

$$e = E\sin\alpha$$

In the above equation α is the angle swept through in t seconds.

Now, 2π is the angle in radians swept through in one cycle;

and $2\pi f$ is the angle in radians swept through in one second;

($2\pi f$ is the angular velocity often designated by the symbol ω)

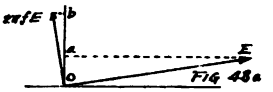
and $2\pi ft$ is the angle in radians swept through in t seconds.

therefore: $\alpha = 2\pi ft$

or: $e = E\sin 2\pi ft$

but: rate of change of $e = \frac{de}{dt} = 2\pi fE\cos 2\pi ft$ (a)

or the rate of change of e can be represented by another uniform



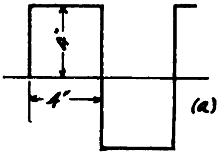
rotating vector (Fig. 48a), which is at each instant proportional to the cos or sine of a uniformly increasing angle 2πft and whose maximum value is equal to 2πf times the maximum value of e, and $\frac{de}{dt}$ is 90° ahead of E (Fig. 48a).

The instantaneous value of E is the vertical projection oa. Likewise the instantaneous value of $\frac{de}{dt}$ is ob. It will be noticed that when E is horizontal, $\frac{de}{dt}$ is a maximum, or the rate of change of any sine wave is a maximum when the wave is passing through zero and is zero when the wave is passing through its maximum position.

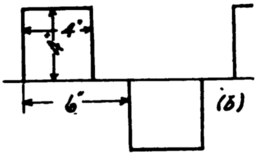
From the calculus we have learned that the rate of change of any curve at a given point is the slope or tangent to the curve at that point.

PROBLEMS.

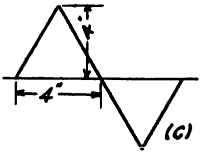
39. Find the form factor in each of the following waves:—



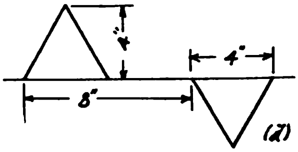
Ans.:—Av. ht. = 4 inches
r.m.s. ht. = 4 inches
f. factor = 1



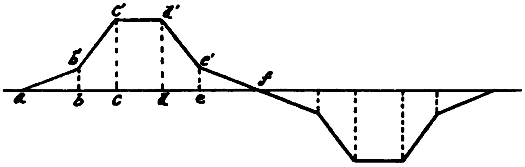
Ans.:—Av. ht. = 2⅔ inches
r.m.s. ht. = 3.27 inches
f. factor = 1.23



Ans.:—Av. ht. = 2 inches
r.m.s. ht. = 2.45 inches (approx.)
f. factor = 1.225



Ans.:—Av. ht. = 1 inch
r.m.s. ht. = 1.73 inch
f. factor = 1.73



ab = 5 inches
bc = 3 inches
cd = 4 inches
de = 3 inches
ef = 5 inches

bb' = ee' = 2 inches
cc' = dd' = 6 inches

Ans.:—Av. ht. = 2.9 inches
r.m.s. ht. = 3.63 inches
f. factor = 1.25

40. An alternating e.m.f. has a frequency of 50 cycles per second; following are the instantaneous values at intervals of .001 second during one half-cycle:—

0 16 30 42 45 46 45 43 32 17 0

Plot the curve and find form factor. *Ans.:*—Av. e.m.f. = 31.6
r.m.s. = 35.0

41. An alternating e.m.f. has a frequency of 50 cycles per second; following are the instantaneous values at intervals of .001 second during one half-cycle:—

0 1.5 2.8 4.0 9.0 15 10 5.0 3.0 1.5

Plot curve and find form factor. How does it compare with f. factor of problem 40?
Ans.:—Av. e.m.f. = 5.18 volts, e. (r.m.s.) = 6.84 volts.

42. The maximum height of a sine wave of e.m.f. is 141 volts. Find (*a*) $e_{(av.)}$, (*b*) $e_{(r.m.s.)}$, (*c*) form factor.

Note:—Make a clock diagram before working any problems.

43. An A.C. generator delivers 100 amperes at 110 volts to incandescent lamps, the angle between the voltage and current is 0°; find the power delivered to the lamps. (When not specified, r.m.s. values are always assumed.) *Ans.:*—11 K.W.
44. If the total resistance of the line is .1 ohm, what is the total power delivered by the generator in problem 43? *Ans.:*—121 K.W.
45. The current and voltage in a given circuit differ by 30°. The current is 600 amperes, the voltage, 1100 volts; find the power. *Ans.:*—571.8 K.W.
46. In an A.C. circuit $E_{(max)} = 141.4$ volts
$I_{(max)} = 707$ amperes
and power factor = .7 Find the power. *Ans.:*—35 K.W.
47. The following readings were taken on an induction motor: $E_{(average)} = 400$ volts, $I_{(average)} = 500$ amperes; the voltage was 45° ahead of the current. (*a*) How many H.P. does the motor receive? (*b*) How many H.P. would the motor receive if the voltage had been 45° behind the current? (*c*) 90° behind? *Ans.:*—(*a*) 233.5 H.P., (*b*) 233.5 H.P., (*c*) —.
48. What is the phase displacement between an $E_{(max)}$ and $I_{(max)}$ respectively of 1000 volts and 100 amperes when the power in the circuit is 42,400 watts? *Ans.:*—64° 58′.
49. Two generators, X and Y, are connected in series. The e.m.f. of X is 2200 volts, and the e.m.f. of Y is 2400 volts. The e.m.f. of X is 90° ahead of the e.m.f. of Y. The two generators give a current of 250 amperes, which lags 30° behind their resultant e.m.f. (*a*) What is their combined e.m.f.? (*b*) What is the power output of each generator? (*c*) What is the p.f. of each generator? (*d*) What is the p.f. of the line? *Ans.:*—(*a*) 3256 volts. (*b*) Output of X, 119,120 watts; output of Y, 585,830 watts. (*c*) –, (*d*) –.
50. The e.m.f. of generator X, problem 49, is 135° ahead of the e.m.f. of generator Y. A current of 240 amperes flows through both generators, lagging 30° behind their resultant e.m.f. What is the output of each generator? *Ans.:*—Output of X, 123,200 watts negative; output of Y, 491,080 watts positive.

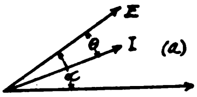
(The negative result means that the machine is running as a motor.)

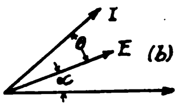
51. A 220-volt generator delivers 100 amperes of current to incandescent lamps in which the voltage and current are in phase with each other, and delivers 37.5 amperes to start a single-phase motor. The p.f. of the

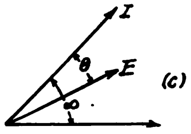
motor while starting is 0.3. (*a*) Find the total current. (*b*) What is the power delivered to the lamps? (*c*) Power delivered to motor? (*d*) Total power? *Ans.:*—(*a*) 116.8 amperes, (*b*) 11.K.W., (*c*) 1237.5 watts, (*d*) 12237.5 watts.

Note:—A.C. generators are sometimes spoken of as alternators.

52. Prove in cases (*a*), (b), and (c) that $P = e_{(r.m.s.)}\, i_{(r.m.s.)} \cos\theta$. In each case draw the sine wave diagram to correspond with the clock diagrams shown. (θ is always the angle between e and i.)

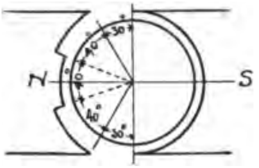






53. The generator shown in the figure has one conductor on its armature and has pole faces shaped as shown. The total flux from the N-pole is 2,000,000 lines and the armature makes 10 revolutions per second. Plot the e.m.f. curve, neglecting fringes of field at corners of pole pieces, and determine average value and r.m.s. value of the e.m.f. during each half-cycle.

The long and short air gaps under the 180° pole face are as 2 to 1. Plot the e.m.f. wave for one complete cycle.



Ans.:—Under the N-pole $\begin{cases} E_{(av.)} = .4 \text{ volt} \\ E_{(r.m.s.)} = .52 \text{ volt} \end{cases}$

Under the S-pole $\begin{cases} E_{(av.)} = .4 \text{ volt} \\ E_{(r.m.s.)} = .4 \text{ volt} \end{cases}$

54. A certain 60-cycle generator delivers a sine wave of current, whose maximum value is 100 amperes, to a receiving circuit. Find the rate of change of the current (*a*) at the instant it passes through zero, (*b*) 30° later, (*c*) 90° later, (*d*) 180° later. *Ans.:*—(*a*) +26,400; (*b*) +22,862.4; (*c*) 0; (*d*) −26,400.

55. In problem 54 the voltage (maximum value 100 volts) is 30° ahead of the current; find the rate of change of voltage at the four points specified in problem 54. *Ans.:*—(*a*) +22,862.4; (*b*) +13,200; (*c*) −13,200; (*d*) −22,862.4.

56. In curve (*e*), problem 39, what is the rate of change from: (*a*) a to b′; (*b*) from b′ to c′; (*c*) from c′ to d′; (*d*) from d′ to e′? (Frequency 25 cycles per second.) *Ans.:*—(*a*) +400; (*b*) +1333⅓; (*c*) 0; (*d*) −1333⅓.

CHAPTER IV.

INDUCTANCE; CAPACITY; SERIES CIRCUITS.

INDUCTANCE.

21. **Inductance.**—From our study of direct currents we learned that a generator had e.m.f. induced in its armature by the cutting of the lines of force emanating from the poles. When current flows in a wire, lines of force encircle the wire. When the circuit is open, the current is zero and the lines of force are zero; and when the circuit is closed and the current increases, the lines of force grow in circles and are thought of as starting from the very center of the wire. These lines growing from the center of the wire cut the wire and therefore induce an e.m.f. Very soon the current reaches a steady value. The lines of force encircling the wire remain constant, the greater number of them being near the region of the wire. (Fig. 49.)

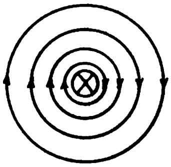


Fig 49

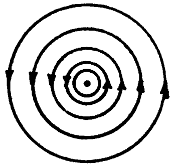


Fig 50

Since the lines remain constant in number, they do not cut the wire, and therefore no e.m.f. is induced.

Now, if the circuit is opened by a switch the current falls to zero, and therefore the lines of force all collapse on the wire and disappear at its center; in doing this they cut the wire, and therefore an e.m.f. is again induced. Fig. 49 shows the direction of the circular lines of force around a conductor which has current flowing away from the reader, and Fig. 50 shows the direction of the circular lines of force around a conductor which has current flowing toward the reader.

The e.m.f. induced in a coil by a changing of the lines of force around that coil is called the e.m.f. of self-induction, and the circular lines of force are spoken of as the magnetic field.

The e.m.f. inducted in a coil S by the changing of the lines of force around an adjacent coil P (Fig. 51) is called the e.m.f. of mutual induction (the e.m.f. induced in the secondary coils of a transformer by the flux induced by the primary).

22. Mechanical Analogy.—Consider a wheel weighing 100 pounds mounted on a shaft and at rest. If you press against a spoke of the wheel, it will begin to revolve. Two forces resist the pressure on the spoke; one due to the friction with the shaft and the other due to the inertia of the wheel. (A 50-pound wheel would have less inertia and therefore would start more quickly than the 100-pound wheel, although its friction with the shaft might be the same.) With a constant pressure against the spoke the wheel would soon reach a constant velocity. During this period friction would be the only force that resists the pressure on the spoke.

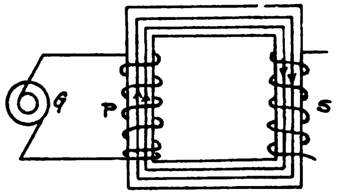


Fig 51

Now, if the pressure on the spoke were suddenly removed, the wheel would not instantly come to rest. The friction would tend to bring it to rest, its momentum due to its weight would tend to keep it revolving, so that it would not come to rest for some time. (A 50-pound wheel would have less momentum and therefore stop more quickly than the 100-pound wheel.) When accelerating, when running with uniform velocity, and when coming to rest, the friction tends to stop the wheel. When starting, its inertia tends to stop the wheel; when running with uniform velocity, its inertia or momentum has no effect; and when coming to rest, its momentum tends to keep it going.

In a direct-current circuit when the switch is closed the current grows to a steady value and the e.m.f. overcomes the electrical friction, called resistance, and the electrical inertia, called inductance (in this case growing lines of force, and if the circuit has less inductance corresponding to the 50-pound wheel, the current will reach its constant value more quickly). Soon the current reaches a constant value, in which case the e.m.f. overcomes resistance only (RI). Now open the switch; the current will not immediately become zero, but dies away slowly, as shown by the spark. The electrical friction (resistance) tends to stop it, and the electrical momentum (inductance) tends to keep it going (in this case collapsing lines of force; and if the circuit has less inductance corresponding to the 50-pound wheel, the current will reach zero more quickly and there will be a smaller spark on opening the switch). Friction and resistance are entirely similar and always oppose the impressed e.m.f. Inertia (or momentum) and inductance are entirely similar and may or may not oppose the impressed e.m.f.

A transmission line has very little inductance. A coil of wire has more inductance and a coil surrounding an iron core has still more. It is because of its greater inductance that opening a field switch of a dynamo carrying but 4 amperes will produce a greater spark than opening a switch on a bank of incandescent lamps carrying 20 or 30 amperes. Because of this high e.m.f. of induction it is dangerous to open any field switch quickly unless there is an alternate path for the current, as the insulation on the field spool may be punctured by the high e.m.f. induced by the suddenly collapsing lines of force.

23. **Inductance (L).**—If a coil of wire of 10 turns and carrying 1 ampere has 10 lines of force threading through, it is said to have 100 linkages. This is called its coefficient of self-induction and is designated by the letter L. In other words, the c.g.s. unit of self-induction L is defined as the number of linkages threading a circuit when one ampere flows through it.

If in the above circuit the current had been 2 amperes, and still there had been but 10 lines of force threading it, then L would be $\frac{10\times10}{2}$;

or letting Φ = total number of lines of force threading the coil
(also spoken of as flux)
and n = total number of turns of wire in the coil
and i = current in c.g.s. units flowing through the circuit

$$\text{then evidently } L=\frac{\Phi\times n}{i} \qquad (11)$$

This equation gives L in c.g.s. units. The c.g.s. unit is too small to use, therefore the practical unit, which equals 10^9 c.g.s. units and is called the henry, is used, and then—

$$L_{(\text{henries})}=\frac{\Phi\times n}{I_{(\text{amperes})}\times 10^8}\qquad \left(1 \text{ ampere}=\frac{1}{10}\text{ c.g.s. unit.}\right)\qquad (12)$$

In direct currents the only time inductance is present is at the instant of closing or opening a circuit, and therefore is neglected; but in alternating currents inductance cannot be neglected because the current is always changing in value. In other words, the only back e.m.f. that has to be overcome by the impressed e.m.f. in direct currents is RI, while in alternating currents the impressed e.m.f. has to overcome two back e.m.f.'s, one due to resistance, as in direct currents, and equal to RI, and the other due to inductance L, and produced by the rising and falling of the lines of force in the circuit.

24. The Work Done Upon a Magnetic Circuit in Starting the Current.—If current (I) be flowing in a circuit with a pressure of (E) volts, then the Power is EI watts and the work done in dt seconds

$$\text{is } d\,w=E\,I\,dt$$

$$L\,I=\Phi\times n \text{ (see equation 12);}$$

$$\text{but } \frac{d\Phi}{10^8\,dt}=\text{volts induced in each turn}$$

$$\frac{n\times d\,\Phi}{10^8\,dt}=E=L\frac{d\,I}{d\,t}$$

Integrating through the full change in value, 0 to W, and 0 to I, we get—

$$W=\int_0^I LI\frac{d\,I}{d\,t}$$

$$W=\tfrac{1}{2}\,L\,I^2 \qquad \text{(expressed in joules)} \qquad (13)$$

Similarly the kinetic energy of the wheel is—

$$W=\tfrac{1}{2}\,K\,\omega^2$$

Where W is the kinetic energy of the revolving wheel, ω is the angular velocity of the wheel, and $\frac{1}{2}$ K is a constant. K is called the moment of inertia of the wheel.

The total voltage in a circuit in which the current is growing is—

$$E=R\,I+L\frac{d\,I}{d\,t} \qquad (14)$$

25. Phase Relations in a Circuit Having Resistance Only.—Consider the A.C. generator G (Fig. 52) connected to the pure resistance R.

Back e.m.f. = RI (Ohm's law) and from the analogy of the wheel the back e.m.f. opposes the flow of current or is 180° from the current (I) (Fig. 53 and Fig. 54). In order that there may be equilibrium, as is always the case in any electrical A.C. or D.C. circuit, the impressed e.m.f. (E) from the generator (Fig. 52) must be equal and opposite to the b.e.m.f. (R I), which means the impressed e.m.f. E = RI and is 180° from b.e.m.f. or is in phase with I (Figs. 53 and 54).

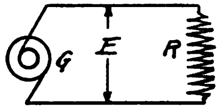


Fig 52

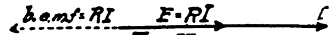


Fig 53

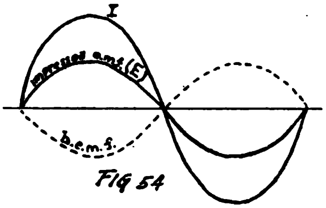


Fig 54

Therefore in a circuit having resistance only, E and I are in phase and E = RI.

26. Phase Relations in a Circuit Having Inductance Only.—Consider the A.C. generator G (Fig. 55) connected to the pure inductance. Fig. 52 is supposed to be a circuit which has so few lines of force linked with it that they may be neglected, as incandescent lamps, and will always be shown as in the figure (pure resistance). Fig. 55 is supposed to be a circuit surrounded by iron and having many lines of force linked with it and having negligible resistance as an induction coil or the field spool of a generator, and will always be shown as in the figure. No device has pure resistance and no inductance or pure inductance and no resistance.

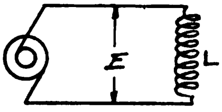


Fig 55

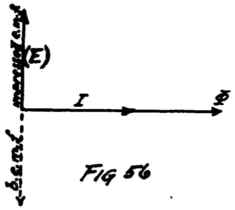


Fig 56

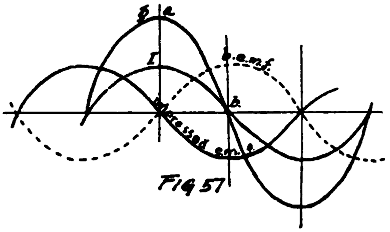


Fig 57

In Fig. 55 when the current is zero the lines of force linked with the circuit are zero, when the current is a maximum the lines of force are a maximum, and when the current is reversed the lines of force are reversed; in other words, the current (I) and the flux Φ are in phase with each other (Figs. 56 and 57). Now, it is this change of flux that produces the b.e.m.f. of self-induction. In Fig. 57 at the point (a) the flux (Φ) is a maximum, and since the wave is horizontal at point (a) the change of flux is zero just at that instant, and therefore the b.e.m.f. at point (a) is zero. At point (b) the flux is changing at a maximum rate, since the Φ curve is steepest or most nearly perpendicular at the point (b), therefore the b.e.m.f. produced by the changing flux is a maximum at the point (b), or the b.e.m.f. is 90° behind Φ, as shown in Fig. 56 and Fig. 57.

In order that there may be equilibrium, as is the case in any A.C. or D.C. circuit, the impressed e.m.f. (E) from the generator (Fig. 55) must be equal and opposite to the b.e.m.f., which means that the impressed e.m.f. (E) is equal to the b.e.m.f. and is 180° from the b.e.m.f., which makes the impressed e.m.f. 90° ahead of Φ and I; see Fig. 56 and Fig. 57.

Therefore in a circuit having inductance only, E is 90° ahead of I.

27. Derivation of Expression for Back e.m.f. Due to Inductance.—From the formula—

$$L=\frac{\Phi n}{I} \qquad \text{(equation 12)}$$

The amount of back e.m.f. induced in any circuit because of inductance depends upon the frequency, upon the amount of current, and upon the coefficient of self-induction, L. If the device has iron in its magnetic circuit, as is generally the case, then L will change for different values of current; for if the iron becomes saturated, then the lines of force will not increase in proportion to the current. In other words, for a given device the coefficient of self-induction L will vary. If, however, the iron is kept below the saturation point, L will not vary.

From the formula—

$$L=\frac{\Phi n}{I} \qquad \text{(equation 12)}$$

Φ = flux corresponding to current I
and n = number of turns with which the flux is linked.

If I is the maximum value of the sine wave of current, then

Φ is the maximum value of the flux wave. Clearly this total flux Φ is cut four times in one cycle:—

once in going from 0 to +maximum
once in going from +maximum back to 0
once in going from 0 to −maximum
once in going from −maximum back to 0

By definition e.m.f. in volts = the number of lines of force cut in one second, divided by 10^8.

$$\text{evidently } 4\,\Phi_{(max)}\,f\,n = \text{flux cut in one second by n turns in series}$$
$$\text{or } E_{(average)} = \frac{4\,\Phi_{(max)}\,f\,n}{10^8}$$

Assuming the induced e.m.f. to be a sine wave—

$$E_{(r.m.s.)} = 4 \times \frac{1.11\,\Phi_{(max)}\,f\,n}{10^8} = 4.44\,\Phi_{(max)}\,f\,n \qquad (15)$$
$$\text{but } L\,I = \Phi\,n \text{ (from equation 12)}$$
$$\text{or } L\,I_{(max)} = \Phi_{(max)} n$$

Substituting $L\,I_{(max)}$ for $\Phi_{(max)}\,n$ in (15), we get—

$$E_{(r.m.s.)} = 4.44\,f\,L\,I_{(max)};$$
$$\text{but } I_{(max)} = I_{(r.m.s.)} \times \sqrt{2}$$
$$\text{therefore } E_{(r.m.s.)} = 4.44\,f\,L\,I_{(r.m.s.)} \times \sqrt{2}$$
$$\text{but } 4.44 \times \sqrt{2} = 4 \times 1.11 \times \sqrt{2} = 4 \times \frac{.707}{.636} \times \sqrt{2} = 4 \times \frac{1}{\sqrt{2}} \times \frac{\pi}{2} \times \sqrt{2} = 2\,\pi$$
$$\text{therefore } E_{(r.m.s.)} = 2\,\pi\,f\,L\,I_{(r.m.s.)} \qquad (16)$$

The impressed e.m.f. to overcome resistance R is $E = RI$, and is in phase with I.

The impressed e.m.f. to overcome inductance L is $E = 2\,\pi\,f\,L\,I$, and is 90° ahead of I.

28. A Calculus Method of Obtaining the Back e.m.f. Due to Inductance ($E = 2\,\pi\,f\,L\,I$).—

$$\Phi\,n = L\,I \qquad \text{(equation 12)}$$
$$\text{but } \frac{n\,d\,\Phi}{10^8\,d\,t} = E_{(volts)}$$
$$E_{(inst)} = e = L\,\frac{d\,I_{(inst)}}{d\,t}$$
$$\text{but } I_{(inst)} = i = I_{(max)} \sin \alpha \qquad (a)$$

$$i = I_{(max)} \sin 2\,\pi\,f\,t \qquad \text{(See art. 20)}$$
$$\frac{d\,i}{d\,t} = 2\,\pi\,f\,I_{(max)} \cos 2\,\pi\,f\,t$$
$$\therefore \quad E_{(inst)} = L\,\frac{d\,i}{d\,t} = 2\,\pi\,f\,L\,I_{(max)} \cos 2\,\pi\,f\,t \qquad (b)$$

The maximum value cos 2 π f t can have, is unity.

$$\therefore \quad E_{(\max)} = 2\,\pi\, f\, L\, I_{(\max)}$$

$$\text{or } \frac{E_{(\max)}}{\sqrt{2}} = E_{(\text{r.m.s.})} = 2\,\pi\, f\, L\, \frac{I_{(\max)}}{\sqrt{2}} = 2\,\pi\, f\, L\, I_{(\text{r.m.s.})} \qquad (16)$$

The fact that equation (a) contains sin α and equation (b) contains cos α shows that E and I are 90° apart.

CAPACITY.

29. Capacity.—If a condenser be placed in a direct-current circuit, the dielectric between the plates acts as an open circuit and will not allow any current to flow. There will be a temporary flow of current at the instant the circuit is closed, due to the charging of the condenser.

If a condenser be placed in an A.C. circuit of 60 cycles, the condenser will be charged, discharged, charged in the opposite direction, and again discharged sixty times every second, so that there will be an alternating current flowing in the line and therefore a condenser will not prevent the flow of A.C.

30. Phase Relations in a Circuit Having Capacity Only.—In order to prove these relations, consider the hydraulic analogy, Fig. 58.

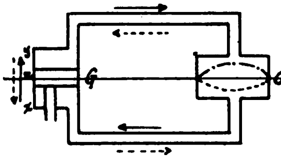


Fig 58

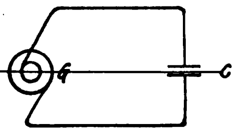


Fig 59

Fig. 59 represents an A.C. generator (G) connected to a condenser (C); Fig. 58 represents a pump (G) connected through a pipe, filled with water, to an elastic diaphragm (C).

Consider the plunger to be moving up, and in the middle of its stroke; then evidently the diaphragm is not distorted, shown by the solid line at C. In other words, the pressure on the diaphragm is zero when the piston is in the middle of the stroke. When the piston is completely elevated, the diaphragm has a maximum positive pressure and is distorted as shown by the dotted line at C. When the piston is completely depressed, the diaphragm has a maximum negative pressure and is distorted as shown by the dot and dash line at C.

If the piston be given a regular alternating up-and-down motion, it will be noticed that the water flows past any point in the pipe circuit in a positive direction, while the piston is going from x to y, and the water will reverse direction at y and flow in a negative direction while the piston is going from y to x.

Piston going up
- When the piston is at z the flow is +maximum: the pressure is zero.
- When the piston is at y the flow is zero: the pressure is +maximum.

Piston going down
- When the piston is at z the flow is −maximum: the pressure is zero.
- When the piston is at x the flow is zero: the pressure is −maximum.

From the analysis it is seen that the pressure produced by the pump is 90° behind the flow of water in the pipe.

Returning to the case of the A.C. generator (G) connected to the condenser (C); the flow of current will be positive from a −maximum of pressure to a +maximum of pressure, and the flow will be a negative from a +maximum of pressure to a −maximum of pressure. This means that the zero values of current occur at the maximum values of pressure, and that the pressure (e.m.f.) produced by the generator is 90° behind the current flowing through the circuit. (See Figs. 60 and 61.)

Following the analogy, it will be noticed that the condenser is completely discharged at point z and completely charged in the opposite direction at point x. The b.e.m.f. produced by the tension in the material of the diaphragm, (C) (Fig. 59), by the tension

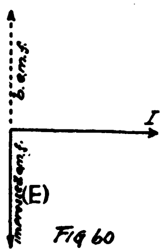


Fig 60

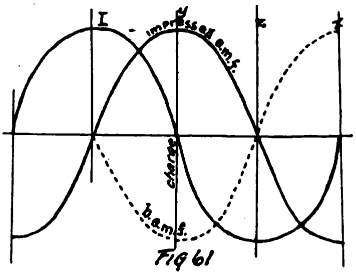


Fig 61

in the dielectric between the plates of the condenser, (C) (Fig. 59), is of course equal and opposite to the impressed e.m.f. (Fig. 60) in order that there may be equilibrium, as is always the case in any A.C. or D.C. circuit, and the b.e.m.f. is therefore 180° from impressed e.m.f., which makes the b.e.m.f. 90° ahead of I. (Figs. 60 and 61.)

Summary.—In the case of pure resistance E and I are in phase.
In the case of pure inductance E is 90° ahead of I.
In the case of pure capacity E is 90° behind I.

31. Derivation of Expression for the Back e.m.f. Due to a Condenser.—By experiment it has been found that the amount of charge (Q) that any condenser will hold is doubled if the e.m.f. impressed on the condenser is doubled; in other words, Q varies directly as E;

$$\text{or } Q = C\,E \tag{17}$$

Where Q is the charge or quantity of electricity on the condenser, E is the e.m.f. impressed on the condenser, and C is a constant and is called the capacity of the condenser. When E is expressed in volts, Q is expressed in coulombs and C is expressed in farads. One farad is a much larger capacity than any practical condenser ever has, so a microfarad, which is .000001 of a farad, is generally the term used, thus:—A 20-microfarad condenser has .000020 farads capacity.

$$\text{Quantity} = \text{velocity or rate of flow} \times \text{time} = v \times t \tag{18}$$

Example:—If electricity flows into a condenser at the rate of 10 amperes (ampere = rate of flow) for ten seconds, the quantity in coulombs = 10 × 10 = 100 coulombs, or

$$Q = I \times t \tag{19}$$

$$\text{Coulombs} = \text{amperes} \times \text{seconds}$$

If the capacity of a condenser be C farads and a maximum e.m.f. $E_{(max)}$ be applied, then evidently the condenser will receive a maximum charge $Q_{(max)} = C\,E_{(max)}$ (C is always a constant).

The condenser is charged to a +maximum, then discharged to O, then charged in the opposite direction to a −maximum, and again discharged to O every cycle or four times; and if the frequency is f cycles per second, then the average rate of charge and discharge is—

4 f times per second, or time of one average charge $=\frac{1}{4f}$ seconds

The maximum rate of charge and discharge = average rate $\times \frac{\pi}{2}$ or

$=4\,f \times \frac{\pi}{2} = 2\,\pi\,f$

Now, maximum charge $Q_{(max)} = C\,E_{(max)}$;

but rate of charge or flow per second $= I = \frac{Q}{t} = \frac{Q}{\frac{1}{2\,\pi\,f}}$ (equation 19);

therefore $I_{(max)} = \frac{Q_{(max)}}{t} = \frac{C\,E_{(max)}}{\frac{1}{2\,\pi\,f}} = 2\,\pi\,f\,C\,E_{(max)}$ (20)

Dividing both sides of equation (20) by $\sqrt{2}$ we have—

$$\frac{I_{(max)}}{\sqrt{2}} = 2\,\pi\,f\,C\,\frac{E_{(max)}}{\sqrt{2}}$$

$$\text{or } I_{(r.m.s.)} = 2\,\pi\,f\,C\,E_{(r.m.s.)} \qquad (21)$$

$$\text{or } E_{(r.m.s.)} = \frac{I_{(r.m.s.)}}{2\,\pi\,f\,C} \qquad (22)$$

32. A Calculus Method of Obtaining the Back e.m.f. Due to a Condenser.—

$$Q = I \times t \qquad \text{(equation 19)}$$

$$I_{(inst)} = \frac{d\,Q_{(inst)}}{dt} \qquad \left(\text{rate of flow} = \frac{\text{Quantity}}{\text{time}}\right)$$

$$\text{but } Q = C\,E$$

$$\therefore \quad \frac{d\,Q_{(inst)}}{dt} = C \times \frac{d\,E_{(inst)}}{dt}$$

$$\text{but } E_{(inst)} = e = E_{(max)} \sin\alpha = I_{(max)} \sin 2\,\pi\,f\,t$$

$$\therefore \quad \frac{d\,Q}{d\,t} = I = C\,\frac{d\,E}{d\,t} = E_{(max)}\,C\,2\,\pi\,f \cos 2\,\pi\,f\,t$$

The maximum value cos $2\,\pi\,f\,t$ can have is unity.

$$\therefore \quad I_{(max)} = E_{(max)}\,2\,\pi\,f\,C$$

$$\text{or } I_{(r.m.s.)} = E_{(r.m.s.)}\,2\pi\,f\,C$$

$$\text{or } E_{(r.m.s.)} = \frac{I_{(r.m.s.)}}{2\,\pi\,f\,C}$$

(For a discussion of the rising and dying currents in circuits having R, L, and C, see Sheldon, Mason, and Hausmann, "Alternating Current Machines," pp. 33–35 and pp. 58 and 59.)

As $W = \frac{1}{2}\,L\,I^2$ (see equation 13)

So can it be proved that the energy required to establish the dielectric stresses when the current is first started is—

$$W = \frac{Q^2}{2\,C} \quad \text{(expressed in joules)} \qquad (23)$$

The impressed e.m.f. to overcome resistance (R) is $E = R\,I$ and is in phase with I.

The impressed e.m.f. to overcome inductance (L) is $E = 2\,\pi\,f\,L\,I$ and is 90° ahead of I.

The impressed e.m.f. to overcome capacity (C) is $E = \frac{I}{2\,\pi\,f\,C}$ and is 90° behind I.

Resistance in a circuit tends to keep E and I in phase.

Inductance tends to make E lead I.

Capacity tends to make E lag behind I and inductance and capacity in the same circuit tend to neutralize each other.

SERIES CIRCUITS.

33. Illustrations of Different Combinations of Series Circuits.—*Case* 1.—A circuit containing resistance and inductance in series:

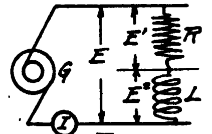


Fig 62

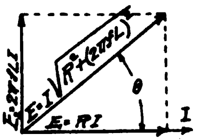


Fig 63

In Fig. 62 let the generator (G) be connected to the pure resistance (R) in series with a pure inductance (L). Since this is a series circuit, there is but one current (I) in all parts of the circuit, and therefore in Fig. 63 I is drawn along the horizontal; E_1 (Fig. 62) is drawn in place with I (Fig. 63) because it is pure resistance; E_2 (Fig. 62) is drawn 90° ahead of I (Fig. 63) because it is pure inductance.

By geometry of Fig. 63 total impressed e.m.f. of the generator—

$$E = \sqrt{(R\,I)^2 + (2\,\pi\,f\,L\,I)^2}$$
$$= I\sqrt{R^2 + (2\,\pi\,f\,L)^2}$$

R is called the resistance, and is expressed in ohms.

$2\,\pi\,f\,L$ is called the reactance, and is expressed in ohms.

$\sqrt{R^2 + (2\,\pi\,f\,L)^2}$ is called the impedance, and is expressed in ohms.

θ is the angle between E and I.

$$\cos\theta = \frac{R\,I}{I\sqrt{R^2 + (2\,\pi\,f\,L)^2}} = \frac{R}{\sqrt{R^2 + (2\,\pi\,f\,L)^2}} = \frac{\text{resistance}}{\text{impedance}}$$

$$\text{power factor} = \cos\theta = \frac{R}{\sqrt{R^2 + (2\,\pi\,f\,L)^2}}$$

$$\sin\theta = \frac{2\,\pi\,f\,L\,I}{I\sqrt{R^2 + (2\,\pi\,f\,L)^2}} = \frac{2\,\pi\,f\,L}{\sqrt{R^2 + (2\,\pi\,f\,L)^2}} = \frac{\text{reactance}}{\text{impedance}}$$

$$\tan\theta = \frac{2\,\pi\,f\,L\,I}{R\,I} = \frac{2\,\pi\,f\,L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

In this case the power factor is positive, as E leads I.

(R I) is the amount of E in phase with I, and is called the power component of E.

(2π f L I) is the amount of E 90° away from I, and is called the wattless component of E.

These two components are obtained by dropping a perpendicular from the end of E on I. (See Fig. 63.)

Case 2.—A circuit containing resistance and capacity in series: In Fig. 64 the generator G is connected to a pure resistance R in series with a condenser or pure capacity C. Again, since this is a series circuit, the current is common, and I (Fig 65) is drawn along the horizontal.

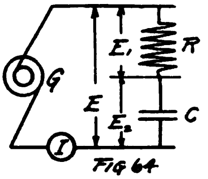


Fig 64

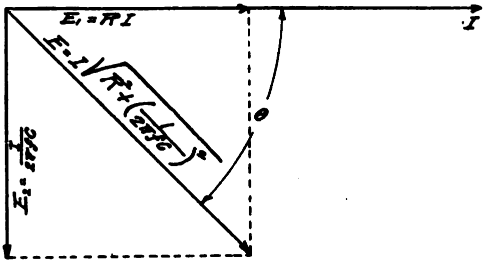


Fig 65

E_1 (Figs. 64 and 65) is in phase with I (pure resistance); E_2 (Figs. 64 and 65) is drawn 90° behind I (pure capacity).

By geometry of Fig. 65, total impressed e.m.f. of the generator—

$$E = \sqrt{(R\,I)^2 + \left(\frac{I}{2\,\pi\,f\,C}\right)^2}$$

$$= I\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}$$

Again, R is resistance in ohms

$\frac{1}{2\,\pi\,f\,C}$ is reactance in ohms

$\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f}\right)^2}$ is impedance in ohms.

$$\text{power factor} = \cos\theta = \frac{R\,I}{I\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,c}\right)^2}} = \frac{\text{resistance}}{\text{impedance}}$$

$$\sin\theta = \frac{\frac{I}{2\,\pi\,f\,C}}{I\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}} = \frac{\frac{1}{2\,\pi\,f\,C}}{\sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}} = \frac{\text{reactance}}{\text{impedance}}$$

$$\tan\theta = \frac{\frac{I}{2\,\pi\,f\,C}}{R\,I} = \frac{\frac{1}{2\,\pi\,f\,C}}{R} = \frac{\text{reactance}}{\text{resistance}}$$

In this case the power factor is negative, as E lags behind I.

Case 3.—A circuit containing inductance and capacity in series: Fig. 66 needs no explanation, since it is a series circuit. I is drawn along the horizontal (Fig. 67).

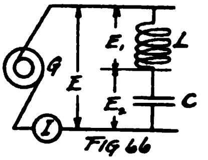


FIG 66

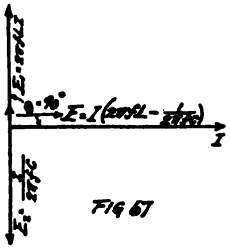


FIG 67

In Fig. 67 E_1 is 90° ahead of I (pure inductance); and E_2 is 90° behind I (pure capacity). Since E_1 and E_2 are 180° apart—

$$E = 2\pi f L I - \frac{I}{2\pi f C} = I\left(2\pi f L - \frac{1}{2\pi f C}\right)$$

$$\text{power factor} = \cos\theta = 0$$
$$\sin\theta = 1$$
$$\tan\theta = \alpha \text{ (infinity)}$$

In Fig. 67, E_1 is supposed to be larger than E_2, and therefore E is 90° ahead of I; if E_2 were larger than E_1, then E would be 90° behind I.

If $E_1 = 104{,}000$ volts and $E_2 = 100{,}000$ volts,
then $E = E_1 - E_2 = 4000$ volts,

or the total voltage E (Fig. 66) across the line may be much less than either branch voltage E_1 or E_2.

If $E_1 = E_2$, then $2\pi f L I = \frac{I}{2\pi f C}$, then the circuit is said to contain *resonance.*

Dividing by I, we get:—

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\text{or } (2\pi f)^2 = \frac{1}{L C}$$

Extracting square root:—

$$2\pi f = \frac{1}{\sqrt{L C}}$$

$$f = \frac{1}{2\pi\sqrt{L C}} \tag{24}$$

The frequency given by equation (24) is called the critical frequency of a circuit, and produces electrical resonance.

Case 4.—A circuit containing resistance, inductance, and capacity in series: Fig. 68 shows all three circuits connected in series.

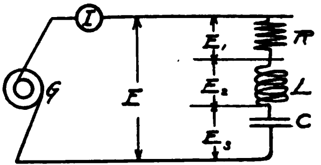
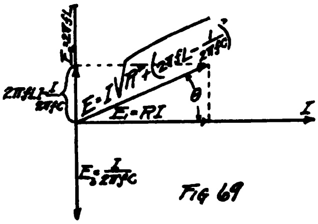


Fig 68


$E_2 = 2\pi fL$
$2\pi fLI - \frac{I}{2\pi fC}$
$E = I\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)}$
$E_1 = RI$
θ
I
$E_3 = \frac{I}{2\pi fC}$
Fig 69

Since I is common (series circuit), it is drawn along the horizontal (Fig. 69).

$E_1 = R\,I$ is in phase with I
$E_2 = 2\,\pi\,f\,L\,I$ is 90° ahead of I.
$E_3 = \dfrac{I}{2\,\pi\,f\,C}$ is 90° behind I.

From the geometry of Fig. 69:—

$$E=\sqrt{\left(RI\right)^2+\left(2\pi fLI-\frac{I}{2\pi fC}\right)^2}=I\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}. \tag{25}$$

power factor = cos θ

$$\cos\theta=\frac{RI}{I\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}=\frac{R}{\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}=\frac{\text{resistance}}{\text{impedance}} \tag{26}$$

$$\sin\theta=\frac{2\pi fLI-\frac{I}{2\pi fC}}{I\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}=\frac{2\pi fL-\frac{1}{2\pi fC}}{\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}=\frac{\text{reactance}}{\text{impedance}} \tag{27}$$

$$\tan\theta=\frac{2\pi fLI-\frac{I}{2\pi fC}}{RI}=\frac{2\pi fL-\frac{1}{2\pi fC}}{R}=\frac{\text{reactance}}{\text{resistance}} \tag{28}$$

In this case E_2 is greater than E_3, therefore E leads I and the power factor is positive.

If E_3 had been greater than E_2, then:—

$$\cos\theta=\frac{R}{\sqrt{R^2+\left(\frac{1}{2\pi fC}-2\pi fL\right)^2}},\text{ and}$$

E would lag behind I and the power factor would be negative.

Case 5.—A circuit containing several R's, L's, and C's in series: In Fig. 70 there are 3 R's, 2 L's, and 2 C's, all connected in series; therefore I is common to the circuit and is drawn along the horizontal in Fig. 71.

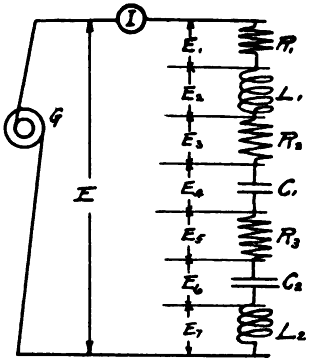


Fig 70

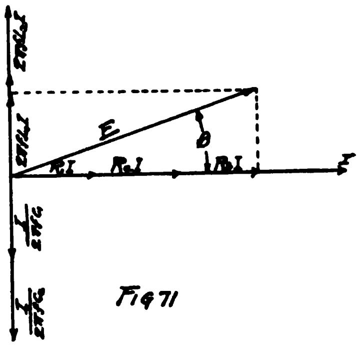


Fig 71

R_1I, R_2I, and R_3I are in phase with I (pure resistance); $2\pi f L_1 I$ and $2\pi f L_2 I$ are 90° ahead of I (pure inductance);

$$\frac{I}{2\pi f C_1} \text{ and } \frac{1}{2\pi f C_2}$$

are 90° behind I (pure capacity)

From the geometry of Fig. 71:—

$$E=\sqrt{\Big[R_1I+R_2I+R_3I\Big]^2+\Big[\Big(2\pi fL_1I+2\pi fL_2I\Big)-\Big(\frac{I}{2\pi fC_1}+\frac{I}{2\pi fC_2}\Big)\Big]^2} \quad (29)$$

$$=I\sqrt{\Big[R_1+R_2+R_3\Big]^2+\Big[\Big(2\pi fL_1+2\pi fL_2\Big)-\Big(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\Big)\Big]^2} \quad (30)$$

power factor $=\cos\theta=$

$$\frac{R_1+R_2+R_3}{\sqrt{\Big[R_1+R_2+R_3\Big]^2+\Big[\Big(2\pi fL_1+2\pi fL_2\Big)-\Big(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\Big)\Big]^2}}=\frac{\text{resistance}}{\text{impedance}}$$

$$\sin\theta=\frac{\Big(2\pi fL_1+2\pi fL_2\Big)-\Big(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\Big)}{\sqrt{\Big[R_1+R_2+R_3\Big]^2+\Big[\Big(2\pi fL_1+2\pi fL_2\Big)-\Big(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\Big)^2}}=\frac{\text{reactance}}{\text{impedance}}$$

$$\tan\theta=\frac{\Big(2\pi fL_1+2\pi fL_2\Big)-\Big(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\Big)}{R_1+R_2+R_2}=\frac{\text{reactance}}{\text{resistance}}$$

In general, in any series circuit consisting of any number of parts—

$$E = I\sqrt{\left[R_1+R_2+\text{etc.}.\right]^2+\left[\left(2\pi fL_1+2\pi fL_2+\text{etc.}.\right)-\left(\frac{1}{2\pi fC_1}+\frac{1}{2\pi fC_2}\text{etc.}.\right)\right]^2} \tag{31}$$

from which cos θ, sin θ, etc., can easily be obtained. Resistance (R) is the quantity which, multiplied by I, gives the amount of E in phase with I (called the power component of E) and is expressed in ohms.

Reactance is the quantity which, multiplied by I, gives the amount of E 90° from I (called the wattless component of E) and is expressed in ohms.

Impedance is the quantity which, multiplied by I, gives E and is compressed in ohms.

REFERENCES.

Oscillations:—See Sheldon, Mason, and Hausmann, "Alternating Current Machines," pp. 82 and 83; also Bedell and Crehore's "Alternating Currents," second edition, pp. 105–109.

Harmonics:—Pender, Principles of Electrical Engineering.

PROBLEMS.

(*Note:*—Make a diagram of connections and a vector, or clock, diagram for each problem.)

(r.m.s. values are always assumed unless otherwise specified.)

57. An alternating e.m.f. of 220 volts is applied to eleven 110-ohm lamps connected in parallel. Assuming that the lamps have resistance only, (*a*). what is the current flowing? (b) what is the phase difference between the e.m.f. and current? (*c*) what is the power factor? *Ans.:*—(*a*) 22 amperes, (*b*) zero, (*c*) unity.
58. Calculate the inductance (L) in henries of one of the field spools of an Edison bi-polar dynamo with the following data: total flux per pole, 500,000 lines, 4750 turns, and an exciting current of .6 ampere. Find the energy required to start the current. *Ans.:*—39.6 henries, 7.13 joules.
59. A 60-cycle A.C. generator is connected to an induction coil (negligible resistance), the inductance of which is .5 henry: (*a*) what e.m.f. is required to send 5 amperes through this circuit? (*b*) what is the power factor? is it positive? *Ans.:*—(*a*) 941.7 volts, (*b*) zero, positive.
60. A 60-cycle generator is connected to a condenser; it is found that a current of 2 amperes flows. The pressure maintained by the generator is 1000 volts. Find the capacity of the condenser. *Ans.:*—5.31 microfarads.
61. A harmonic alternating current, maximum value 200 amperes and frequency 120 cycles, flows through a circuit consisting of a non-inductive resistance of 4 ohms, a resistanceless inductance of .006 henry, and a condenser of which the capacity is .00012 farad, all connected in series. Draw a clock diagram representing the phases of the e.m.f.'s. (*a*) across the resistance, (*b*) across the inductance, (*c*) across the condenser, and calculate the r.m.s. value of each. *Ans.:*—(*a*) 565.6 volts, (*b*) 639.8 volts, (*c*) 1563 volts.
62. In problem 61 find (*a*) the total voltage of the generator. (*b*) What is the

power factor in each part of the circuit? (*c*) What is the power factor of the generator? *Ans.:*—(*a*) 1064.7 volts, (*b*) unity, zero, zero, (*c*) .531 (about 57° –54′).

63. A circuit has an inductance of .4 henry and a resistance of 12 ohms. (*a*) Calculate the value of the current produced by an e.m.f. of 200 volts; (*b*) calculate the phase difference between the e.m.f. and current. *Ans.:*—(*a*) 1.32 amperes, (*b*) 85° 27′.

64. (*a*) Calculate the reactance in ohms of .06 henry at 120 cycles.
  (*b*) Calculate the reactance in ohms of a 10 microfarad condenser at 120 cycles.
  (*c*) Find the reactance of both connected in series.
  (*d*) If a resistance of 10 ohms is connected in series with these two and all three in series are connected to a 110-volt generator, calculate the voltage across each part of the circuit.
  (*e*) Calculate the total voltage across all three circuits.
  (*f*) At what frequency is the reactance of both in series equal to zero?
  (*g*) Calculate the voltage across each part of the circuit at the critical frequency.
  (*h*) Calculate the total voltage across all three circuits at the critical frequency.
  (*i*) What is the power factor at the critical frequency?
  Ans.:—(*a*) 45.2 ohms; (*b*) -132.6 ohms; (*c*) -87.4 ohms; (*d*) 56.6 volts across L, 166 volts across C, 12.5 volts across R; (*e*) 110; (*f*) 205.4; (*g*) 850.4 across L and C, 110 across R; (*h*) 110; (*i*) unity.

65. A generator giving 60 cycles per second is connected to a circuit having a resistance of 12 ohms and an inductance of .2 henry. (*a*) What current will flow in the circuit if the generator maintains a pressure of 500 volts? (*b*) What will be the value of the angle of lag? (*c*) What is the power factor? (*d*) Find the power used in the circuit. *Ans.:*—(*a*) 6.55 amperes, (*b*) 80° 57′, (*c*) .172, (*d*) 563.5 watts.

66. (*a*) What would the current have been in problem 64 (*d*) if the circuit had been of negligible resistance? (*b*) What would be the angle of lag in this case? *Ans.:*—(*a*) 1.259 amperes, (*b*) E lags behind I by 90°.

67. (*a*) Find the current produced by a 60-cycle e.m.f. of 240 volts in a circuit having 20 ohms resistance and an inductance of .08 henry. (*b*) What is the power factor and is it positive? If θ is the angle between E and I, find (*c*) $\sin \theta$, (*d*) $\tan \theta$, (*e*) power, (*f*) power component of E, (*g*) wattless component of E, (*h*) power component of I, (*i*) wattless component of I. *Ans.:*—(*a*) 6.63, (*b*) $+.552$, (*c*) .829, (*d*) 1.51, (*e*) 878.4, (*f*) 132.6 volts, (*g*) 200.2 volts, (*h*) 3.66 amperes, (*i*) 5.5 amperes.

68. What should the inductance, problem 65, be to attain a power factor of .894? *Ans.:*—.0265 henry.

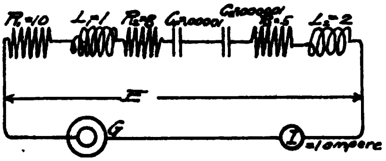
69. Derive an expression for the current in a circuit (*a*) which has a resistance equal to its reactance, (*b*) which has a capacity reactance equal to its inductive reactance.

70. A harmonic e.m.f. of 2200 volts produces a harmonic current of 200 amperes in a given circuit, the current being 20° behind the e.m.f. Calculate the (*a*) resistance, (*b*) reactance, (*c*) impedance of the circuit. *Ans.:*—R = 10.34 ohms, (*b*) reactance ($2\pi f L$) = 3.76 ohms, (*c*) 11 ohms.

71. A harmonic e.m.f. of 220 volts produces a current of 30 amperes in a receiving circuit of which the power factor is $+.7$. Find the values (*a*) resistance, (*b*) reactance, (*c*) impedance. (*d*) Does the current lead or lag behind the voltage and how much? *Ans.:*—(*a*) 5.13 ohms, (*b*) $+5.24$ ohms, (*c*) 7.33 ohms, (*d*) lags 45° 36′.

72. (*a*) Find the voltage across each part of circuit shown on top of p. 49.
  (*b*) Find the total E
  (*c*) Find the power factor.
  (*d*) Is E ahead of I?
  Ans.:—(*a*) 10, 100, 8, 1000, 10000, 5, 200, (*b*) 107,000.2, (*c*) .00215, (*d*) behind I.

73. In an induction motor the voltage leads the current 45°. If the motor takes 100 amperes at 220 volts, find (*a*) power, (*b*) impedance, (*c*) reactance, (*d*) resistance, (*e*) sin θ, (*f*) cos θ, (*g*) tan θ. *Ans.*:—(*a*) 15554 watts, (*b*) 2.2, (*c*) 1.55, (*d*) 1.55, (*e*) .707, (*f*) .707, (*g*) unity.



74. In a series circuit having only R and L the total voltage is 220 volts, current 80 amperes, and the power factor .8; find (*a*) voltage in phase with current, (*b*) voltage 90° ahead of current. *Ans.*:—(*a*) 176, (*b*) 132.

75. In problem 70 find (*a*) current in phase with E, (*b*) current 90° behind E. *Ans.*:—(*a*) 188, (*b*) 68.4.

76. In a series circuit having R and L, the total voltage is 220 volts, the current 100 amperes, the resistance 2 ohms. Find (*a*) impedance, (*b*) reactance, (*c*) the power factor. *Ans.*:—(*a*) 2.2, (*b*) .916, (*c*) .909.

77. A transmission line of which the resistance is 40 ohms and the reactance is 30 ohms, delivers 100 amperes to a non-inductive receiving circuit, the electromotive force across the terminals of the receiving circuit being 100,000 volts. (*a*) What is the generator voltage? (*b*) What is the phase difference between generator voltage and the voltage across the receiving circuit? *Ans.*:—(*a*) 104,043 volts, (*b*) 1° 39′.

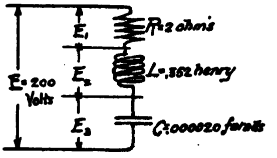
78. A practically non-inductive transmission line having a resistance of 40 ohms delivers 100 amperes of current to a certain receiving circuit. The phase-difference between the voltage and current in the receiving circuit is practically 90°; the voltage across the terminals of the receiving circuit is 100,000 volts. What is the generator voltage? *Ans.*: —100,080 volts.

79. If the transmission line of problem 77 were to deliver 100 amperes of current at 100,000 volts to a condenser, what would be the generator voltage? *Ans.*:—97,031 volts.

80. The transmission line of problem 77 delivers 100 amperes of current at 100,000 volts to a receiving circuit which has a power factor of 0.7. What is the value of the generator voltage. *Ans.*:—104,946 volts.

81. A transmission line which has a resistance of 10 ohms and a reactance of 12 ohms delivers current to a receiving circuit. The receiving circuit consists of a resistance of 400 ohms and a reactance of 300 ohms connected in series. Voltage at receiving circuit is 10,000. Find (*a*) voltage at generator, (*b*) p.f. at receiving circuit, (*c*) p.f. at generator. *Ans.*:—(*a*) 10,304, (*b*) .8, (*c*) .796.

82. A 30-pole 100 r.p.m. generator supplies 100 amperes at 200 volts to an induction motor the p.f. of which is .85. What is the inductance of the motor in henries? *Ans.*: —.1676 henry.



83. In the circuit shown (*a*) find the critical frequency; (*b*) find E_1, E_2, and E_3, at the critical frequency; (*c*) find power factor at critical frequency. (*d*) Calculate the current at 20, 40, 60, 80, and 100 cycles and plot a curve, amperes as ordinates and cycles as abscissæ. *Ans.*:—(*a*) 60, (*b*) 200, 13270, 13270, (*c*) unity.

CHAPTER V.

PARALLEL CIRCUITS.

34. Illustrations of Different Kinds of Parallel Circuits.—*Case* 1.—A circuit consisting of R and L in series, connected in parallel with a circuit consisting of R and C in series (see Fig. 72):

In the series circuit (x) the current (I_1) is common; therefore in the clock diagram (Fig. 73), I_1 is drawn horizontal, E_1 is in phase with I_1 and E_2 is 90° ahead of I_1, and E, the voltage of the generator, is $\theta_1°$ ahead of I_1.

In the series circuit (y) the current (I_2) is common; therefore in clock diagram (Fig. 74), I_2 is drawn horizontal, E_3 is in phase with I_2, and E_4 is 90° behind I_2. In this case E, the voltage of the generator, is $\theta_2°$ behind I_2.

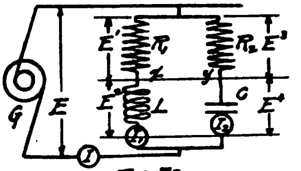


FIG 72

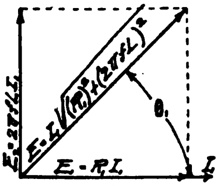


FIG 73

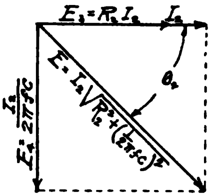


FIG 74

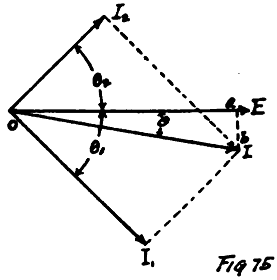


FIG 75

Fig. 75 is a combination of Figs. 73 and 74. Since x and y are in parallel, E is common, and is therefore drawn horizontal.

The E of Fig. 75 is, of course, the E of Fig. 73, being parallel circuits, and in Fig. 73 if E be drawn horizontal, I_1 will take the position shown in Fig. 75. E_1 and E_2 are left out of Fig. 75 to avoid complexity.

Again, the E of Fig. 75 is the E of Fig. 74, and in Fig. 74 if E be drawn horizontal, I_2 will take the position shown in Fig. 75. E_3 and E_4 are not drawn in Fig. 75.

From Fig. 75 and from equation (3) we have:—

$$I=\sqrt{(oa)^2+(ab)^2}$$

$$I=\sqrt{(I_1\cos\theta_1+I_2\cos\theta_2)^2+(I_1\sin\theta_1-I_2\sin\theta_2)^2} \qquad \text{(a)}$$

from Figs. 72 and 73:—

$$\cos\theta_1=\frac{R_1}{\sqrt{R_1^2+(2\pi fL)^2}} \qquad \sin\theta_1=\frac{2\pi fL}{\sqrt{R_1^2+(2\pi fL)^2}}$$

from Figs. 72 and 74:—

$$\cos\theta_2=\frac{R_2}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}} \qquad \sin\theta_2=\frac{\frac{1}{2\pi fC}}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}$$

Substituting these values in equation (a) we have:—

$$I=\sqrt{\left[I_1\frac{R_1}{\sqrt{R_1^2+\left(2\pi fL\right)^2}}+I_2\frac{R_2}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}\right]^2+\left[I_1\frac{2\pi fL}{\sqrt{R_1^2+\left(2\pi fL\right)^2}}-I_2\frac{\frac{1}{2\pi fC}}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}\right]^2}$$

from Fig. 75:—

$$\text{power factor}=\cos\theta=\frac{oa}{I}=\frac{I_1\cos\theta_1+I_2\cos\theta_2}{I}$$

$$=\frac{I_1\dfrac{R_1}{\sqrt{R_1^2+\left(2\pi fL\right)^2}}+I_2\dfrac{R_2}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}}{I}$$

In Fig. 75 the power factor is positive, since E leads I by $\theta°$. If both circuits x and y had consisted of R and L in series, then I_2 and I_1 would both be behind E, and, of course, I would also lag behind E.

Case 2.—Fig. 76 represents the connections for case 2.

In the series circuit x there is one current, I_1, which is therefore drawn along the horizontal (Fig. 77) in phase with one voltage, E_1, which $= E$.

In the series circuit y, there is only one current, I_2, which is therefore drawn along the horizontal (Fig. 78), and E_2 is in phase with I_2 (pure R.) and E_3 is 90° ahead of I_2 (pure L.).

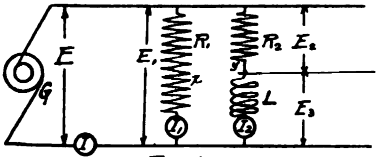


Fig 76

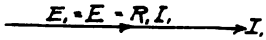


Fig 77

The E of Fig. 77 is the E of Fig. 78 (parallel circuits), and is therefore drawn along the horizontal (Fig. 79).

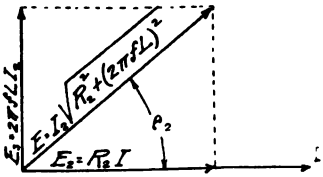


Fig 78

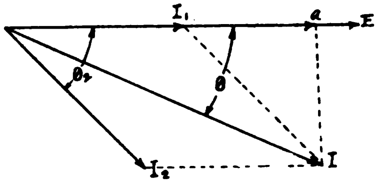


Fig 79

I_1 is in phase with E (Fig. 79) and I_2 is behind E by the angle θ_2 (see Fig. 78).

from Fig. 77:—

$$\theta_1 = 0° \qquad \text{and } \cos\theta_1 = 1 \qquad \text{and } \sin\theta_1 = 0$$

from Fig. 78:—

$$\cos\theta_2 = \frac{R_2}{\sqrt{R_2^2+(2\pi f L)^2}} \qquad \text{and } \sin\theta_2 = \frac{2\pi f L}{\sqrt{R_2^2+(2\pi f L)^2}}$$

$$\therefore I = \sqrt{\left[\left(I_1 \times 1\right)+\left(I_2\frac{R_2}{\sqrt{R_2^2+(2\pi f L)^2}}\right)\right]^2+\left[\left(I_1 \times 0\right)+\left(I_2\frac{2\pi f L}{\sqrt{R_2^2+(2\pi f L)^2}}\right)\right]^2}$$

$$= \sqrt{\left[I_1+I_2\frac{R_2}{\sqrt{R_2^2+(2\pi f L)^2}}\right]^2+\left[I_2\frac{2\pi f L}{\sqrt{R_2^2+(2\pi f L)^2}}\right]^2}$$

from Fig. 79:—

$$\text{power factor} = \cos\theta = \frac{oa}{I} = \frac{I_1+I_2\frac{R_2}{\sqrt{R_2^2+(2\pi f L)^2}}}{I}$$

Case 3.—Fig. 80 represents three circuits, x, y, and z, connected in parallel.

In circuit x, E, and I_1 are in phase (Fig. 81).
In circuit y, E is 90° ahead of I_2 (Fig. 82).
In circuit z, E is θ_3° behind I_3 (Fig. 83).

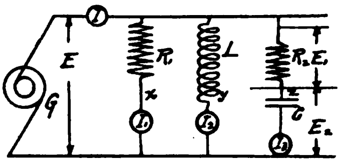


Fig 80

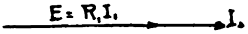


Fig 81

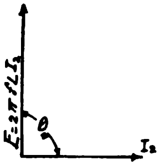


Fig 82

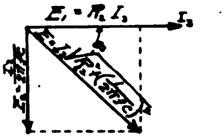


Fig 83

E is common to all three circuits, and is therefore drawn along the horizontal (Fig. 84) and I_1, I_2, and I_3 take the positions shown from Figs. 81, 82, and 83.

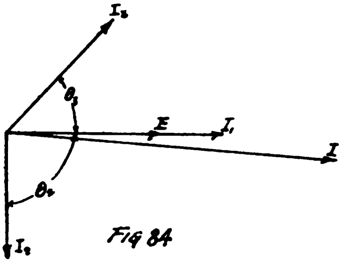


Fig 84

From Fig. 81, $\theta_1=0$.
From Fig. 82, $\theta_2=90°$.
From Fig. 83, θ_3 is as shown.
From Fig. 84:—

$$I=\sqrt{\left(I_1\cos\theta_1+I_2\cos\theta_2+I_3\cos\theta_3\right)^2+\left(I_1\sin\theta_1+I_2\sin\theta_2-I_3\sin\theta_3\right)^2}$$

$$=\sqrt{\left(I_1+I_3\frac{R_2}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}\right)^2+\left(I_2-I_3\frac{\frac{1}{2\pi fC}}{\sqrt{R_2^2+\left(\frac{1}{2\pi fC}\right)^2}}\right)^2}$$

In general in any number of parallel circuits—

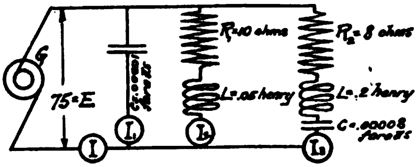
$$I = \sqrt{(I_1 \cos\theta_1 + I_2 \cos\theta_2 + \text{etc.})^2 + (\pm I_1 \sin\theta_1 \pm I_2 \sin\theta_2 + \text{etc.})^2} \quad (32)$$

$$\text{and power factor} = \frac{I_1 \cos\theta_1 + I_2 \cos\theta_2 + \text{etc.}}{I} \quad (33)$$

All I sin θ's above the horizontal should be added together, and this sum subtracted from the sum of all I sin θ's below the horizontal; the result will tell whether the total I is ahead or behind E. If this result comes out negative, it does not matter, as the quantity has to be squared.

PROBLEMS.

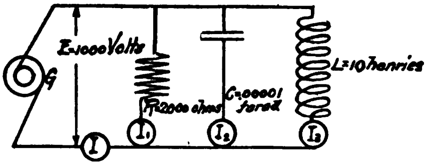
84. In Fig. 72 assume $E=100$, $2\pi f=100$, $R_1=14.1$ ohms, $L=.141$ henry, $R_2=14.1$ ohms, and $C=.000707$ farads.
find (*a*) θ_1, (*b*) θ_2, (*c*) I_1, (*d*) I_2, (*e*) p.f. *Ans.*:—(*a*) –, (*b*) –, (*c*) –, (*d*) .707 amperes, (*e*) unity.



85. In the 50-cycle circuit shown in the figure find:—(*a*) I_1, (*b*) I_2, (*c*) I_3, (*d*) θ_1, (*e*) θ_2, (*f*) θ_3, (*g*) I, (*h*) p.f. (Assume $2\pi f=314$.)
Ans.:—(*a*) –, (*b*) –, (*c*) –, (*d*) –90°, (*e*) +57° 30′, (*f*) +70° 49′, (*g*) 6.84 amperes, (*h*) .464 ($\theta=62°\ 22'$).

86. Draw the clock diagram for problem 85 carefully to scale. Determine the angles θ_1, θ_2, and θ_3, using a protractor, draw I_1, I_2, and I_3, and completing the figure find I and power factor.
(*Note:*—Most parallel circuits are worked graphically.)

87. In the figure shown assume $2\pi f=100$: find (*a*) I_1, (*b*) I_2, (*c*) I_3, (*d*) p.f. of each circuit, (*e*) I, (*f*) p.f. of generator. *Ans.*:—(*a*) $I_1=.5$, (*b*) $I_2=1$, (*c*) $I_3=1$, (*d*) $\begin{cases}1\\0,\\0\end{cases}$ (*e*) .5, (*f*) 1.



(*Note:*—This problem shows that the total current I may be less than one of the branch currents, I_2 or I_3. This parallel circuit has resonance and the critical frequency is $\frac{100}{2\pi}=15.9$ cycles, since $2\pi f=100$.)

88. A generator delivers 200 amperes to circuit A at a p.f. of +.8; it delivers

300 amperes to circuit B at a p.f. of +.6. If circuits A and B are connected in parallel; (*a*) what is the total current and (*b*) the p.f. of the combination. *Ans.*:—(*a*) 495.2 amperes, (*b*) .686.
(*Note:*—When the p.f. has no sign before it, thus .7, it is always assumed to be +.)

89. If a 60-cycle 2200-volt generator delivers 50 amperes to a motor having a p.f. of .9, what should be the capacity of a condenser placed in parallel with the motor to just compensate for the lagging current, and thus make the p.f. of the transmission line unity? What is the total current delivered by the generator? *Ans.*:—26.25 microfarads, (*b*) 45 amperes.

CHAPTER VI.

THE COMPLEX QUANTITY.

In chapter I two vectors were added graphically by completing the parallelogram with the two vectors as sides; and two vectors were subtracted graphically by reversing the one to be subtracted and completing the parallelogram. It is impossible to multiply or divide two vectors graphically.

35. **Representation of Vectors.**—Consider the vector P (Fig. 85) to be in the first quadrant as shown.

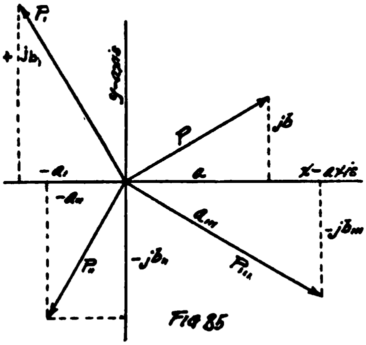


Fig 85

P is made up of two components, an x-component (*a*) and a y-component (*b*). In order to distinguish the x- and y-components from each other Steinmetz puts the letter j in front of the y-component so that the vector P may be represented thus:—

$$P = a + jb$$

Clearly the numerical value of P is:

$$P = \sqrt{a^2 + b^2}$$

The vector P in the second quadrant is represented thus:—

$$P_{,} = -a_{,} + jb_{,}$$

The x-component in this case is to the left and therefore negative.

The vector $P_{,,}$ in the third quadrant is represented thus:—

$$P_{,,} = -a_{,,} - jb_{,,}$$

The x-component in this case is to the left and therefore negative; the y-component in this case is vertically downward and therefore negative.

Likewise any vector in the fourth quadrant is represented thus:—

$$P_{,,,} = a_{,,,} - jb_{,,,}$$

36. **Example.**—In Fig. 86 the two vectors, $P_{,}$ and $P_{,,}$, both in the first quadrant, are to be (1) added, (2) subtracted, (3) multiplied, (4) divided.

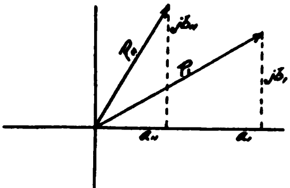


Fig 86

1. *Addition:*—

$$P_{,} = a_{,} + jb_{,}$$
$$\underline{P_{,,} = a_{,,} + jb_{,,}}$$

By algebra:— $$P_{,} + P_{,,} = a_{,} + a_{,,} + j\,(b_{,} + b_{,,}) \tag{34}$$

The vector which represents $P_{,} + P_{,,}$ has an

x-component $a_{,} + a_{,,}$
and a y-component $b_{,} + b_{,,}$

The numerical value of the sum is:—

$$P_{,} + P_{,,} = \sqrt{(a_{,} + a_{,,})^2 + (b_{,} + b_{,,})^2}$$

2. *Subtraction:*—

$$P_{,} = a_{,} + jb_{,}$$
$$\underline{P_{,,} = a_{,,} + jb_{,,}}$$

By algebra:— $$P_{,} - P_{,,} = a_{,} - a_{,,} + j\,(b_{,} - b_{,,}) \tag{35}$$

The vector which represents $P_{,} - P_{,,}$ has an

x-component $a_{,} - a_{,,}$
and a y-component $b_{,} - b_{,,}$

The numerical value of this difference is:—

$$P_{,} - P_{,,} = \sqrt{(a_{,} - a_{,,})^2 + (b_{,} - b_{,,})^2}$$

3. *Multiplication:*—

$$P_{,}=a_{,}+jb_{,}$$
$$P_{,,}=a_{,,}+jb_{,,}$$

By algebra:— $P_{,}\times P_{,,}=a_{,}\,a_{,,}+ja_{,,}\,b_{,}+ja_{,}\,b_{,,}+j^2\,b_{,}\,b_{,,}$ (a)

but— $j^2=-1$ (will be proved)

therefore (a) becomes:—

$$P_{,}\times P_{,,}=a_{,}\,a_{,,}-b_{,}\,b_{,,}+j\,(a_{,,}\,b_{,}+a_{,}\,b_{,,}) \qquad (36)$$

The vector which represents $P_{,}\times P_{,,}$ has an

x-component $a_{,}\,a_{,,}-b_{,}\,b_{,,}$
and a y-component $a_{,,}\,b_{,}+a_{,}\,b_{,,}$

The numerical value of this product is:—

$$P_{,}\times P_{,,}=\sqrt{(a_{,}\,a_{,,}-b_{,}b_{,,})^2+(a_{,,}b_{,}+a_{,}b_{,,})^2}$$

4. *Division:*—

$$P_{,}=a_{,}+jb_{,}$$
$$P_{,,}=a_{,,}+jb_{,,}$$

By algebra:— $\frac{P_{,}}{P_{,,}}=\frac{a_{,}+jb_{,}}{a_{,,}+jb_{,,}}$

multiply both numerator and denominator by $a_{,,}-jb_{,,}$ and we have:—

$$\frac{P_{,}}{P_{,,}}=\frac{a_{,}+jb_{,}}{a_{,,}+jb_{,,}}\times\frac{a_{,,}-jb_{,,}}{a_{,,}-jb_{,,}}=\frac{a_{,}\,a_{,,}+ja_{,,}\,b_{,}-ja_{,}\,b_{,,}-j^2\,b_{,}\,b_{,,}}{a_{,,}^2-j^2\,b_{,,}^2}$$

substituting $j^2=-1$ we have:—

$$\frac{P_{,}}{P_{,,}}=\frac{a_{,}\,a_{,,}+j\,(a_{,,}\,b_{,}-a_{,}\,b_{,,})+b_{,}\,b_{,,}}{a_{,,}^2+b_{,,}^2}$$

re-arranging the j-terms together we have:—

$$\frac{P_{,}}{P_{,,}}=\frac{a_{,}\,a_{,,}+b_{,}\,b_{,,}}{a_{,,}^2+b_{,,}^2}+\frac{j\,(a_{,,}\,b_{,}-a_{,}\,b_{,,})}{a_{,,}^2+b_{,,}^2} \qquad (37)$$

by multiplying both numerator and denominator by the denominator with the sign in front of the j quantity changed we are always able to get a quantity free from j, in this case:—

$$\frac{a_{,}\,a_{,,}+b_{,}\,b_{,,}}{a_{,,}^2+b_{,,}^2}$$

and a purely j−quantity, in this case:—

$$\frac{a_{,,}\,b_{,}-a_{,}\,b_{,,}}{a_{,,}^2+b_{,,}^2}$$

The vector which represents $\frac{P_{,}}{P_{,,}}$ has an

$$\text{x-component } \frac{a_{,}a_{,,}+b_{,}b_{,,}}{a_{,,}^2+b_{,,}^2}$$

$$\text{and a y-component } \frac{a_{,,}b_{,}-a_{,}b_{,,}}{a_{,,}^2+b_{,,}^2}$$

The numerical value of this division is:—

$$\frac{P_{,}}{P_{,,}}=\sqrt{\left(\frac{a_{,}a_{,,}+b_{,}b_{,,}}{a_{,,}^2+b_{,,}^2}\right)^2+\left(\frac{a_{,,}b_{,}-a_{,}b_{,,}}{a_{,,}^2+b_{,,}^2}\right)^2}$$

37. **Numerical Example.**—Consider the two vectors, $P_{,}$ and $P_{,,}$, Fig. 87.

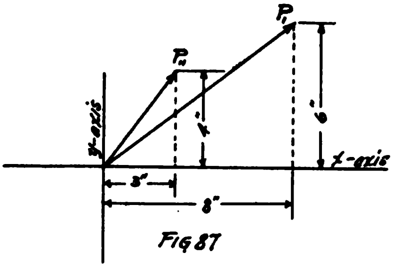


FIG 87

As before, $P_{,}=a_{,}+jb_{,}$ and $P_{,,}=a_{,,}+jb_{,,}$, since both are in the first quadrant. From the figure:—

$a_{,}=8$ inches $b_{,}=6$ inches
and $a_{,,}=3$ inches $b_{,,}=4$ inches

1. *Adding:*—

$$P_{,}+P_{,,}=a_{,}+a_{,,}+j(b_{,}+b_{,,}) \text{ See equation.} \qquad (34)$$

Substituting the values of $a_{,}$, $a_{,,}$, $b_{,}$ and $b_{,,}$ in equation (34) we get:—

$$\begin{aligned}P_{,}+P_{,,}&=(8+3)+j\,(6+4)\\&=11+j\,(10)\end{aligned}$$

or the sum of $P_{,}+P_{,,}$ has an x-component 11 inches to the right
and a y-component 10 inches vertically
upward,

which makes $P_{,}+P_{,,}$ fall in the first quadrant.

Numerically:—

$$P_{,}+P_{,,}=\sqrt{(11)^2+(10)^2}=14.86 \text{ inches}$$

This result can be obtained graphically by completing the parallelogram as suggested in chapter I (see Fig. 88).

2. *Subtracting:*—

$$P_{,} - P_{,,} = a_{,} - a_{,,} + j(b_{,} - b_{,,}) \quad \text{See equation.} \qquad (35)$$

Substituting:—

$$\begin{aligned} P_{,} - P_{,,} &= (8'' - 3'') + j\ (6'' - 4'') \\ &= 5'' + j\ (2'') \end{aligned}$$

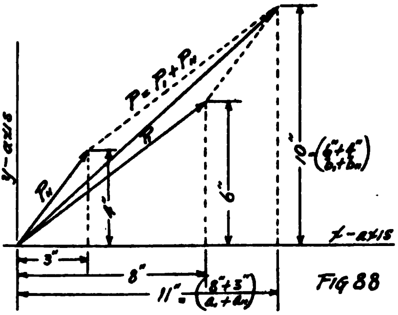


Fig 88

In this case the x-component is 5 inches to the right and the y-component is 2 inches vertically upward, which makes $P_{,} - P_{,,}$ fall in the first quadrant.

Numerically:—

$$P_{,} - P_{,,} = \sqrt{(5)^2 + (2)^2} = 5.38''$$

This result can be obtained graphically by reversing $P_{,,}$, since it is to be subtracted, and complete the parallelogram as suggested in chapter I. (See Fig. 89.)

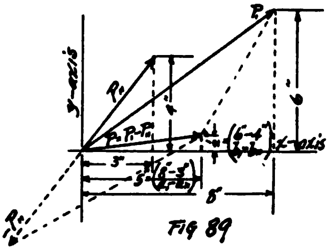


Fig 89

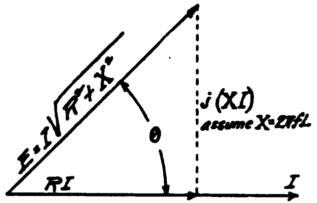


Fig 90

3. *Multiplying:*—

$$P_{,} \times P_{,,} = a_{,}\ a_{,,} - b_{,}\ b_{,,} + j\ (a_{,,}\ b_{,} + a_{,}\ b_{,,}) \quad \text{See equation.} \qquad (36)$$

Substituting:—

$$P_{,} \times P_{,,} = [(8'' \times 3'') - (6'' \times 4'')] + j\ [(3'' \times 6'') + (8'' \times 4'')]$$
$$P_{,} \times P_{,,} = (24'' - 24'') + j\ (50'') = 0 + j\ (50)$$

$P_{,} \times P_{,,} = j\ (50)$. This is a vector 50 inches long and vertically upward.

The numerical value of:—

$$P_{,} \times P_{,,} = \sqrt{(0)^2 + (50)^2} = 50$$

4. *Division:*—

$$\frac{P_{,}}{P_{,,}}=\frac{a_{,}\,a_{,,}+b_{,}\,b_{,,}}{a_{,,}^{2}+b_{,,}^{2}}+j\,\frac{(a_{,,}\,b_{,}-a_{,}\,b_{,,})}{a_{,,}^{2}+b_{,,}^{2}}\qquad \text{See equation. (37)}$$

Substituting:—

$$\frac{P_{,}}{P_{,,}}=\frac{24+24}{9+16}+j\left(\frac{18-32}{9+16}\right)=\frac{48}{25}-j\left(\frac{14}{25}\right)$$

This is a vector in the fourth quadrant, since the x-component $\frac{48}{25}$ is to the right and the y-component $\frac{14}{25}$ is vertically downward.

The numerical value of:—

$$\frac{P_{,}}{P_{,,}}=\sqrt{\left(\frac{48}{25}\right)^{2}+\left(\frac{14}{25}\right)^{2}}=2\ \text{inches.}$$

38. Series Circuits.—Consider any series circuit; since I is common to the circuit, it is drawn as the reference or x-axis (Fig. 90). Assume E to be ahead of I by θ degrees. Drop a perpendicular from the end of E at right angles to I and thus get the two components of E.

The component of E in phase with I = RI. R = resistance in ohms.

The component of E 90° from I = XI. X = reactance in ohms.

and $$E=\sqrt{(R^{2}+X^{2})}\ I. \quad \sqrt{R^{2}+X^{2}}=\text{impedance in ohms} \qquad (38)$$

Using the complex quantity, we have:—

$$E=(R+jX)\,I. \quad R+jX=\text{impedance in ohms}$$

Or numerically $E=(\sqrt{R^{2}+X^{2}})I$, which is identical with equation (38).

R is the quantity by which I is multiplied to get the amount of E in phase with I.

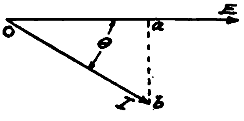


Fig 91

X is the quantity by which I is multiplied to get the amount of E 90° away from I.

R+jX is the quantity by which I is multiplied to get E and numerically = $\sqrt{R^{2}+X^{2}}$.

39. Parallel Circuits.—Consider any parallel circuit; since E is common to the circuit, it is drawn as the reference or x-axis (Fig. 91).

Assume I to be behind E by θ degrees. (E leads I in Figs. 90 and 91.)

Drop a perpendicular from the end of I at right angles to E and thus get the two components of I, o a and a b.

The component of I in phase with E = o a (Fig. 91).

but $\quad o\,a = I\cos\theta$

but $\quad I=\frac{E}{\sqrt{R^2+X^2}}$ and $\cos\theta=\frac{R}{\sqrt{R^2+X^2}}$

therefore $\quad o\,a=\frac{E}{\sqrt{R^2+X^2}}\times\frac{R}{\sqrt{R^2+X^2}}=\left(\frac{R}{R^2+X^2}\right)E$

or the component of I in phase with $E=\left(\frac{R}{R^2+X^2}\right)E$. $\frac{R}{R^2+X^2}=$ conductance in mhos (generally called g)

The component of I 90° from E = a b (Fig. 91).

but $\quad a\,b = I\sin\theta$

but $\quad I=\frac{E}{\sqrt{R^2+X^2}}$ and $\sin\theta=\frac{X=(2\pi fL)}{\sqrt{R^2+X^2}}$

therefore $\quad a\,b=\frac{E}{\sqrt{R^2+X^2}}\times\frac{X}{\sqrt{R^2+X^2}}=\left(\frac{X}{R^2+X^2}\right)E$

or the component of I 90° away from $E=\left(\frac{X}{R^2+X^2}\right)E$. $\frac{X}{R^2+X^2}=$ susceptance in mhos (generally called b)

and $$I=\sqrt{\left(\frac{R}{R^2+X^2}E\right)^2+\left(\frac{X}{R^2+X^2}E\right)^2}=E\sqrt{\frac{R^2}{(R^2+X^2)^2}+\frac{X^2}{(R^2+X^2)^2}}$$

$$=E\sqrt{\frac{R^2+X^2}{(R^2+X^2)^2}}=E\frac{1}{\sqrt{R^2+X^2}}.\quad \frac{1}{\sqrt{R^2+X^2}}=\text{admittance in mhos}\qquad(39)$$

Note:—The mhos is the unit of measurement for conductance, susceptance, and admittance, and is the reciprocal of the ohm: 5 ohms = $\frac{1}{5}$ mhos.

Using the complex quantity,

$$E=(R+j\,X)\,I$$

therefore

$$I=\frac{E}{R+j\,X}$$

Multiplying both numerator and denominator by R − j X we have:—

$$I=E\frac{R-j\,X}{R^2+X^2}=E\left[\frac{R}{R^2+X^2}-j\left(\frac{X}{R^2+X^2}\right)\right]=E\,(g-jb)$$

$\frac{R}{R^2+X^2}$ (conductance) is the quantity by which E is multiplied to get the amount of I in phase with E.

$\frac{X}{R^2+X^2}$ (susceptance) is the quantity by which E is multiplied to get the amount of I 90° from E.

$\left[\frac{R}{R^2+X^2}-j\left(\frac{X}{R^2+X^2}\right)\right]$ (admittance) is the quantity by which E is multiplied to get I and is numerically equal to:—

$$\sqrt{\left(\frac{R}{R^2+X^2}\right)^2+\left(\frac{X}{R^2+X^2}\right)^2}=\sqrt{\frac{R^2+X^2}{(R^2+X^2)^2}}=\frac{1}{\sqrt{R^2+X^2}}. \quad \text{See equation (39).}$$

Impedance $=\sqrt{R^2+X^2}$, therefore admittance $=\frac{1}{\text{impedance}}$

In the equation $E=(R+j\ X)\ I$.

If $2\pi fL$ is greater than $\frac{1}{2\pi fC}$, then E is ahead of I and $E=(R+j\ X)\ I$.

If $2\pi fL$ is less than $\frac{1}{2\pi fC}$, then E is behind I and $E=(R-j\ X)\ I$.

In both Figs. 90 and 91 E is ahead of I, therefore we use the complex equation $E=(R+j\ X)\ I$ and not $E=(R-j\ X)\ I$.

In any complex equation, as $E=R\ I\pm j\ X\ I$,

RI is called the $\left\{\begin{array}{l}\text{real}\\ \text{rational}\\ \text{power}\end{array}\right\}$ component of E

j X I is called the $\left\{\begin{array}{l}\text{imaginary}\\ \text{irrational}\\ \text{wattless}\end{array}\right\}$ component of E

and R I+j X I is called a complex quantity.

$$E=(R+j\ X)\ I \qquad I=(g-jb)\ E \qquad g-jb=\frac{1}{R+j\ X}$$

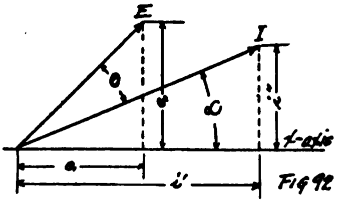


Fig 92

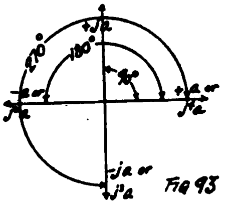


Fig 93

40. Meaning of j.—In Fig. 92 assume E is $\theta°$ ahead of I and let I be $\alpha°$ ahead of the x-axis:—

Now $E=I\ (R+j\ X)$

but $I=i_{,}+j\ i_{,,}$

therefore $E=(i_{,}+j\ i_{,,})\ (R+j\ X)$

$=i_{,}\ R+j\ i_{,,}\ R+j\ i_{,}\ X+j^2\ i_{,,}\ X$ (40)

Again $E=e_{,}+j\ e_{,,}$

but $e_{,}=E\cos(\alpha+\theta)=E\ (\cos\alpha\cos\theta-\sin\alpha\sin\theta)$ (a)

and $e_{,,}=E\sin(\alpha+\theta)=E\ (\sin\alpha\cos\theta+\cos\alpha\sin\theta)$ (b)

Substituting for $E = I\sqrt{R^2+X^2}$ and the values of cos α, ccs θ, sin α, sin θ in (a) and (b)

we get $e_{,} = I\sqrt{R^2+X^2}\left(\frac{i_{,}}{I}\frac{R}{\sqrt{R^2+X^2}} - \frac{i_{,,}}{I}\frac{X}{\sqrt{R^2+X^2}}\right) = i_{,}R - i_{,,}X$

Similarly $e_{,,} = i_{,,}R + i_{,}X$

therefore $E = e_{,} + j\,e_{,,} = i_{,}R - i_{,,}X + j\,(i_{,,}R + i_{,}X)$ (41)

Equating equations (40) and (41)

we get $i_{,}R + j\,(i_{,,}R + i_{,}X) + j^2 i_{,,}X = i_{,}R - i_{,,}X + j\,(i_{,,}R + i_{,}X)$

or $j^2 i_{,,}X = -i_{,,}X$

or $j^2 = -1$ (42)

$j = \sqrt{-1}$ which is an imaginary quantity.

We know that a positive sign before a quantity means along the horizontal to the right. Put a j before the quantity and it revolves the quantity vertically upward or 90° counter-clockwise. Put a − before a quantity and it means the quantity is revolved 180° or is along the horizontal to the left.

But $j^2 = -1$

therefore j^2 rotates a quantity 180°

j^3 rotates a quantity 270°, etc. (See Fig. 93.)

41. Summary.—

R = resistance, since

$g = \frac{R}{R^2+X^2}$ $R = g\,(R^2+X^2) = \frac{g}{g^2+b^2}$ Unit = ohm.

X = reactance, since

$b = \frac{X}{R^2+X^2}$ $X = b\,(R^2+X^2) = \frac{b}{g^2+b^2}$ Unit = ohm.

Z = impedance

$Z = \sqrt{R^2+X^2}$ or $(R \pm j\,X)$ Unit = ohm.

Total impedance

$= \sqrt{(R_{,}+R_2+\text{etc.})^2+(X_{,}+X_2+\text{etc.})^2}$

and total E (any series circuit)

$= I\sqrt{(R_{,}+R_2+\text{etc.})^2+(X_{,}+X_2+\text{etc.})^2}$. See equation (31). (43)

g = conductance

$= \frac{R}{R^2+X^2}$ unit $= \frac{1}{\text{ohm}} =$ mhos

b = susceptance

$= \frac{X}{R^2+X^2}$ unit $= \frac{1}{\text{ohm}} =$ mhos

y = admittance

$= \sqrt{g^2+b^2}$ or $(g \mp jb) = \frac{1}{\sqrt{R^2+X^2}} = \frac{1}{Z}$ unit $= \frac{1}{\text{ohm}} =$ mhos

Total admittance

$= \sqrt{(g_{,}+g_2+\text{etc.})^2+(b_{,}+b_2+\text{etc.})^2}$.

Total I (any parallel circuit)

$= E\sqrt{(g_{,}+g_2+\text{etc.})^2+(b_{,}+b_2+\text{etc.})^2}$. Sec equation (32). (44)

In series circuits it is more convenient to use resistance, reactance, and impedance, as they are additive (equation 43).

In parallel circuits it is more convenient to use conductance, susceptance, and admittance, as they are additive (equation 44).

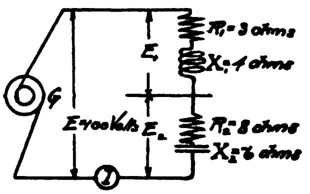


Fig 94

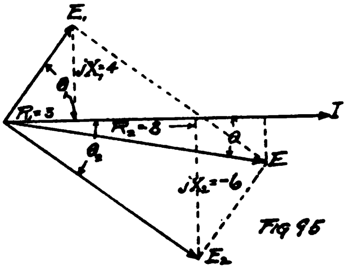


Fig 95

42. Example of a Series Circuit.—Consider the series circuit (Fig. 94) made up of two parts, E_1 and E_2, with the clock diagram to correspond (Fig. 95).

$$E = [(R_1+R_2)+j\,(X_1+X_2)]\,I = [3+8+j\,(4-6)]\,I = [11-j2]\,I$$

$$\text{or } I = \frac{E}{11-j\,2} = \frac{100}{\sqrt{(11)^2+(2)^2}} = 8.94 \text{ amperes (and I is ahead of E)}$$

$$E_1 = (R_1+jX_1)\,I = (3+j\,4)\,8.94 = (\sqrt{(3)^2+(4)^2})\,8.94 = 44.7 \text{ volts}$$

$$E_2 = (R_2+jX_2)\,I = (8-j\,6)\,8.94 = (\sqrt{(8)^2+(6)^2})\,8.94 = 89.4 \text{ volts}$$

$$\tan\theta = \frac{2}{11} \text{ or } \theta = 10^\circ\,20' \text{ and power factor} = \cos 10^\circ\,20' = .968$$

$$P_{(total)} = E\,I\cos\theta = 100\times 8.94\times .968 = 867 \text{ watts}$$

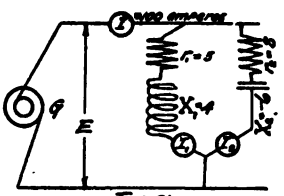


Fig 96

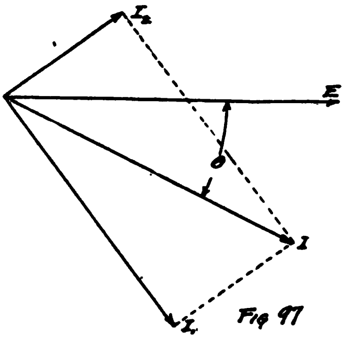


Fig 97

Similarly cos θ_1, cos θ_2, P_1, and P_2 may be found.

43. Example of a Parallel Circuit.—Let the two circuits of Fig. 94 be connected in parallel as in Fig. 96 with the clock diagram to correspond (Fig. 97).

$$I=[(g_{,}+g_2)+j\ (b_{,}+b_2)]\ E$$

$$g_{,}=\frac{R_{,}}{R_{,}^2+X_{,}^2}=\frac{3}{9+16}=\frac{3}{25}\qquad g_2=\frac{R_2}{R_2^2+X_2^2}=\frac{8}{64+36}=\frac{8}{100}=\frac{2}{25}$$

$$b_{,}=\frac{X_{,}}{R_{,}^2+X_{,}^2}=\frac{4}{9+16}=\frac{4}{25}\qquad b_2=\frac{X_2}{R_2^2+X_2^2}=\frac{-6}{64+36}=\frac{-6}{100}=\frac{-3}{50}$$

$$\therefore\ I=\left[\left(\frac{3}{25}+\frac{2}{25}\right)+j\left(\frac{8}{50}-\frac{3}{50}\right)\right]E=\left[\frac{10}{50}+j\left(\frac{5}{50}\right)\right]E \text{ (and E is ahead of I)}$$

$$\text{or } E=\frac{I}{\frac{2}{10}+j\frac{1}{10}}=\frac{100}{\sqrt{(.2)^2+(.1)^2}}=\frac{100}{.224}=447 \text{ volts}$$

$$I_{,}=(g_{,}+jb_{,})\ E=\left(\frac{3}{25}+j\frac{4}{25}\right)447=\left[\sqrt{\left(\frac{3}{25}\right)^2+\left(\frac{4}{25}\right)^2}\right]447=89.4 \text{ amperes}$$

$$I_2=(g_2+jb_2)\ E=\left(\frac{4}{50}-j\frac{3}{50}\right)447=\left[\sqrt{\left(\frac{4}{50}\right)^2+\left(\frac{3}{50}\right)^2}\right]447=44.7 \text{ amperes}$$

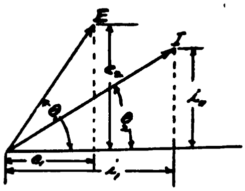
$$\tan\theta=\frac{\frac{5}{50}}{\frac{10}{50}}=.5 \text{ or } \theta=26^\circ\ 30' \text{ and power factor}=\cos 26^\circ\ 30'=.801$$

$$P_{(total)}=E\ I\cos\theta=447\times 100\times .801=358 \text{ watts}$$

Similarly $\cos\theta_{,}$, $\cos\theta_2$, $P_{,}$, and P_2 may be found.

PROBLEMS.

90. Work problem 83 using the complex quantity method.
91. Work problem 72 using the complex quantity method.
92. Work problem 85 using the complex quantity method.
93. Work problem 87 using the complex quantity method.



94. In the figure shown prove that:—
    (*a*) Real power $=e_{,}\ i_{,}+e_2\ i_2$
    (*b*) Reactive power $=e_{,}\ i_2-e_2\ i_{,}$
95. From problem 94 derive expression for:—
    (*a*) Power factor $[\cos\ (\theta_{,}-\theta_2)]$
    (*b*) $\tan\ (\theta_{,}-\theta_2)$
96. (*a*) In a series circuit prove that real power $=R\ I^2$.
    (*b*) In a series circuit prove that reactive power $=X\ I^2$.
97. (*a*) In a parallel circuit prove that real $P=g\ E^2$.
    (*b*) In a parallel circuit prove that reactive $P=b\ E^2$.
    (*Note:*—The expressions in problems 96 and 97 are true for either series, parallel or simple circuits, but those in problem 96 are generally used in series and those in 97 in parallel circuits.)
98. Show that (*a*) $R\ I^2=g\ E^2$
    (*b*) $X\ I^2=b\ E^2$

CHAPTER VII.

SINGLE-PHASE, TWO-PHASE, AND THREE-PHASE RELATIONS.

44. Single-phase.—As stated in chapter II, a single-phase current may be produced by fastening the two ends of a D.C. armature winding to rings and taking off current without rectifying it with a commutator.

It may also be produced by a winding shown in Fig. 98. This is a single-phase four-pole generator. When the armature is in the position shown, the e.m.f. is a maximum because the conductors are directly under the middle of the poles and are therefore cutting the flux at the maximum rate.

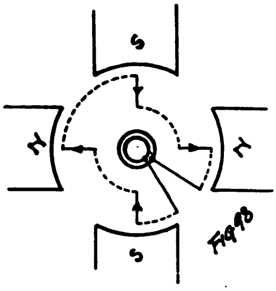


Fig 98

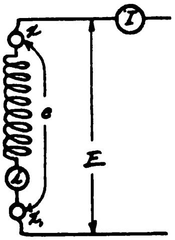


Fig 99

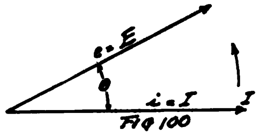


Fig 100

Fig. 98 is represented graphically by Fig. 99, which represents any single-phase circuit.

Throughout the chapter e and i represent phase e.m.f.'s and phase currents, and E and I represent line e.m.f.'s and line currents (Fig. 99).

Evidently $e = E$
and $i = I$ (the phase and line are in series)
But power $P = e\ i \cos\theta$
where θ is the angle between e and i

Substituting E for e, and I for i

we get $$P = E\ I \cos\theta \qquad (45)$$

Fig. 100 is the clock diagram corresponding to Fig. 99, and is self-explanatory.

45. Two-phase Four-wire.—Fig. 101 represents a two-phase four-wire system. Phase A is shown by the full lines and phase B by the dotted lines. Each phase has two collector rings, shown by the four circles at the center. There is no electrical connection between the two phases. At the instant shown in Fig. 101 phase A has a maximum e.m.f. and phase B has zero e.m.f.

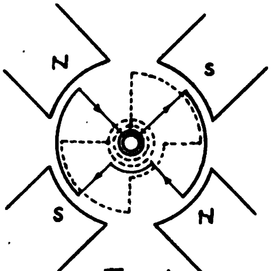


Fig 101

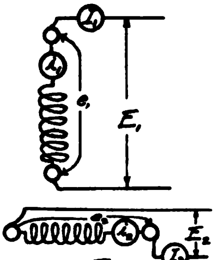


Fig 102

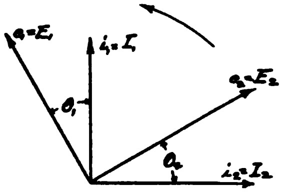


Fig 103

Fig. 101 is represented graphically by Fig. 102.

Evidently from Fig. 102:—

$$e_1 = E_1 \qquad e_2 = E_2$$
$$i_1 = I_1 \qquad i_2 = I_2$$

But Power in phase A $\quad P_1 = e_1\, i_1 \cos \theta_1$.

Power in phase B $\quad P_2 = e_2\, i_2 \cos \theta_2$.

Substituting the values for e_1, i_1, e_2, and i_2

we get $$P_1 = E_1\, I_1 \cos \theta_1 \tag{46}$$
and $$P_2 = E_2\, I_2 \cos \theta_2 \tag{47}$$

where θ_1 is the angle between e_1 and i_1 and

θ_2 is the angle between e_2 and i_2

If each phase has the same p.f. and the same load, as is generally the case, then—

$$E_1 = E_2,\ I_1 = I_2, \text{ and } \theta_1 = \theta_2$$
and $$P_{(\text{total})} = P_1 + P_2 = E_1I_1\cos\theta_1 + E_2I_2\cos\theta_2 = 2E_1I_1\cos\theta_1 = 2EI\cos\theta \tag{48}$$

Fig. 103 is the clock diagram corresponding to Fig. 102, and is self-explanatory.

46. Two-phase Three-wire.—Consider the same generator as shown in Fig. 101 and connect either ring of phase A to either ring in phase B. (See Fig. 104.)

Evidently $i_1 = I_1$
and $i_2 = I_2$ (the line and phase are in series)

Evidently $e_1 = E_1$
and $e_2 = E_2$ (the line and phase are in parallel)

In order to obtain I and E in terms of phase currents and voltages it is necessary to make some arbitrary choice of signs.

To obtain I:—The current in the middle line wire (I) is alternating and we will consider it to be positive when it is flowing away from the generator, as shown by the arrow, and of course it will be negative when it flows toward the generator. Clearly the current in the middle wire is made up of the two-phase currents i_1 and i_2. In order to determine whether it is their sum or their difference it is necessary to give an arbitrary choice of signs to i_1 and i_2. We will assume that i_1 is considered positive when flowing toward the junction o, as indicated by the arrow (Fig. 104), and we will assume that i_2 is positive when it flows toward the junction o, as indicated by the arrow (Fig. 104).

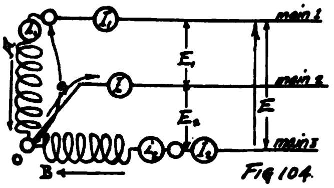


Fig 104.

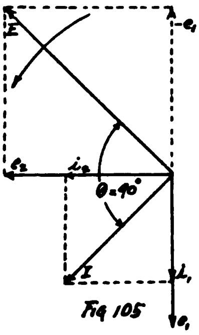


Fig 105

With this choice of signs it is clear that

$$I = i_1 + i_2$$

Notice carefully that in Fig. 105 i_1 and i_2 are drawn exactly corresponding to Fig. 104, and since

$$I = i_1 + i_2$$

we obtain I in Fig. 105 by completing the parallelogram.

For simplicity we will assume a balanced load:—

then $\quad i_1 = i_2$ and $e_1 = e_2$

and by geometry $\quad I = \sqrt{2}\, i_1 = \sqrt{2}\, i_2 = \sqrt{2}\, i$

To obtain E:—The e.m.f. across the outside wires E is, of course, alternating, and we will consider it to be positive when it is pushing from main 3 toward main 1, as shown by the arrow, Fig. 104. (It will, of course, be negative when pushing from main 1 toward main 3.) Clearly E is made up of e_1 and e_2. In order to determine whether it is their sum or their difference it is necessary arbitrarily to give a choice of signs to e_1 and e_2. We will assume that e_1 is considered positive when pushing from main 1 toward o just the same as for i_1, and as indicated by the arrow (Fig. 104); we will also assume e_2 is positive when pushing from main 3 toward o just the same as for i_2, and as shown by the arrow. With this choice of signs from Fig. 104 we can see that

$$E = e_2 - e_1$$

Assume unity power factor, then e_1 and i_1 are in phase, and e_2 and i_2 are in phase; therefore e_1 will fall on i_1 and e_2 will fall on i_2 (Fig. 105).

Since $$E = e_2 - e_1$$

we can obtain E by reversing e_1 and completing the parallelogram as explained in chapter I.

Since $e_1 = e_2$

therefore $E = \sqrt{2}\, e_1 = \sqrt{2}\, e_2 = \sqrt{2}\, e$

$E_1 = e_1$

$E_2 = e_2$

and, still more important, Fig. 105 shows that I and E are 90° apart.

It is for this reason that if a wattmeter has its current coil placed in main 2 and the current (I) passes through it, and has its pressure coil placed between mains 1 and 3 so that it has an e.m.f. (E), then the wattmeter will clearly record:—

$$E\ I \cos\theta$$

where θ is the angle between E and I (see Fig. 105)

but θ is 90° and $\cos\theta = 0$

therefore the wattmeter reads $E \times I \times 0 = 0$

Throughout the two-phase three-wire discussion an arbitrary choice of signs was used. Any choice of signs which you may choose always gives the same result, and therefore it makes no difference, in any polyphase (two or more phases) problem, which direction is chosen as positive and which as negative. It is always easier, and therefore safer, for a student to make a choice of signs which is more or less systematic, and also to put the letters on the clock diagram with as much system as possible.

47. To Prove the Relations in a Two-phase Three-wire System with Another Arbitrary Choice of Signs Different from Those on Fig. 104.—Assume e_1 and i_1 to be positive up from o, and assume e_2 and i_2 positive toward o, and E and I positive as shown by the arrows (Fig. 106).

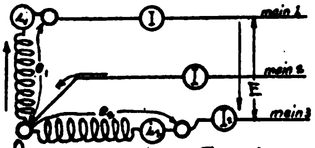


Fig 106

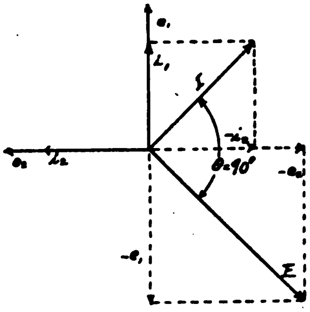


Fig 107

Fig. 107 is a clock diagram representing the choice of signs shown on Fig. 106.

From Fig. 106 evidently

$$I = i_1 - i_2$$

Reversing i_2 and completing the parallelogram we get I as shown (Fig. 107) and by geometry:—

$$I = \sqrt{2}\, i_1 = \sqrt{2}\, i_2 = \sqrt{2}\, i$$

with this choice of signs (Fig. 106) we get:—

$$E = -e_1 - e_2$$

or reversing e_1 and e_2 and completing the parallelogram (Fig. 107) we get E as shown and by geometry:—

$$E = \sqrt{2}\, e_1 = \sqrt{2}\, e_2 = \sqrt{2}\, e$$

Compare closely Figs. 105 and 107 and note the following:—

1. In both figures I is 90° ahead of E.
2. In both figures e_1 and i_1 are 45° ahead of I.
3. In both figures I is 45° ahead of e_2 and i_2.
4. In both figures e_2 and i_2 are 45° ahead of E.
5. In both figures E is 45° ahead of e_1.

Thus we see that any choice of signs always gives the correct numerical value and the correct phase relation. Fewer vectors had to be reversed in Fig. 105 than in Fig. 107, which means that the choice of signs used on Fig. 104 is simpler and therefore better than that used on Fig. 106.

$$\text{Power}_{\text{(per phase)}} = e_1\, i_1 \cos\theta_1$$

where θ_1 is the angle between e_1 and i_1;

but $e_1 = E_1$ and $i_1 = I_1$

therefore $P_{\text{(per phase)}} = E_1 I_1 \cos\theta_1$ (evident from Fig. 104)

but $e_1 = \dfrac{E}{\sqrt{2}}$ and $i_1 = I_1$

therefore $P_{\text{(per phase)}} = \dfrac{E}{\sqrt{2}} I_1 \cos\theta_1$

but $e_1 = E_1$ and $i_1 = \dfrac{I}{\sqrt{2}}$

therefore $P_{\text{(per phase)}} = E_1 \dfrac{I}{\sqrt{2}} \cos\theta_1$

but $e_1 = \dfrac{E}{\sqrt{2}}$ and $i_1 = \dfrac{I}{\sqrt{2}}$

therefore $P_{\text{(per phase)}} = \dfrac{E}{\sqrt{2}} \dfrac{I}{\sqrt{2}} \cos\theta_1 = \dfrac{E\,I}{2} \cos\theta_1$

but if the load be balanced, the power in each phase is the same, and $\theta_1 = \theta_2 = \theta$

$$P_{\text{(total)}} = 2\,P_{\text{(per phase)}} = \sqrt{2}\,E\,I_1 \cos\theta \qquad (49)$$

$$= \sqrt{2}\,E_1\,I \cos\theta \qquad (50)$$

$$= E\,I \cos\theta \qquad (51)$$

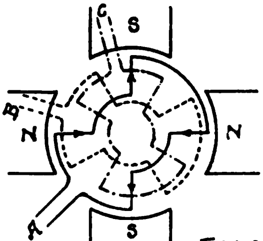


Fig 108

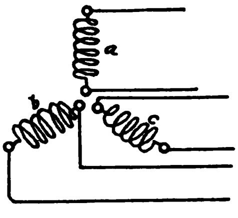


Fig 107

or the total power in a two-phase three-wire circuit = the e.m.f. across the outside wires × current in middle wire × power factor of either phase (with motor load they are generally the same, that is, $\theta_1 = \theta_2$).

48. **Three-phase Six-wire.**—Fig. 108 represents a four-pole

three-phase generator. It is seen that the left-hand sides of each phase are 120° apart, and that this generator has six rings and that there is no electrical connection between the three phases.

Fig. 109 shows the conditions in Fig. 108 graphically.

Fig. 110 is the clock diagram representing Figs. 108 and 109. In Fig. 110 (a) is 120° ahead of (c) and (c) is 120° ahead of (b).

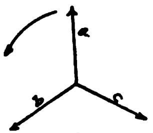


Fig 110

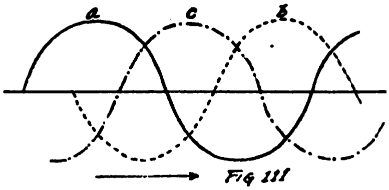


Fig 111

Fig. 111 is the sine wave diagram obtained from 110. (Such a diagram could be readily drawn from Figs. 100 and 103.)

Clearly in this case e = E and i = I

and $$P_{(total)} = 3\ P_{(per\ phase)} = 3\ E\ I\ \cos\theta$$

49. **Three-phase Four-wire.**—The great objection to the three-phase six-wire system is the large number of wires required for distribution.

Fig. 112 shows the next three-phase development, and in this case two of the wires of Fig. 109 are done away with. Assuming a symmetrical choice of signs as shown on Fig. 112, it is seen that the current flowing in main 4 is equal to a+b+c; but a, b, and c, are 120° apart, and from Fig. 113 a +b +c = o

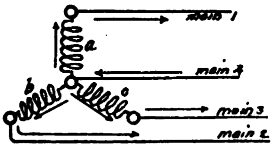


Fig 112

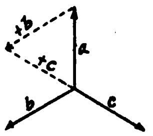


Fig 113

For this reason it was found that main 4 never carried any current, and therefore was not needed.

Power = 3 E I cos θ, just as in the three-phase six-wire circuit.

50. **Three-phase Y- (or Star) connected System.**—Fig. 112 with the useless middle wire removed reduces itself to Fig. 114, which because it looks like the capital Y is called a three-phase Y-connected system.

As in the single- and two-phase cases, small letters denote phase quantities and large letters line quantities.

Since each line is in series with a phase, the phase currents equal the line currents,

that is $$i = I$$

It therefore remains to establish the relations between the phase e.m.f.'s and the line e.m.f.'s or between e and E.

For this purpose assume a symmetrical choice of signs, that is, that the phase e.m.f.'s are considered positive when pushing away from the junction o. Further assume that the line e.m.f.'s (E) are considered positive when pushing from one main to another in a counter-clockwise direction (the large arrows Fig. 114).

Evidently the line e.m.f.'s are made up of two-phase e.m.f.'s connected in series in each case. (See Fig. 114.)

The first is made up of phases e_1 and e_2, and therefore called E_3.

Similarly E_2 is made up of e_1 and e_3

and E_1 is made up of e_2 and e_3

∴ Fig. 114 is symmetrically lettered.

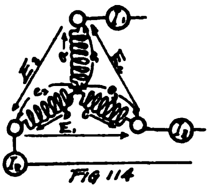


Fig 114

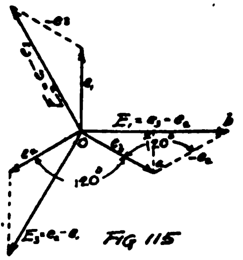


Fig 115

Now, e_1, e_2, and e_3 are 120° apart, as shown in Fig. 115. From the choice of signs given in Fig. 114

$$E_1 = e_3 - e_2$$
$$E_2 = e_1 - e_3$$
and $$E_3 = e_2 - e_1$$

Fulfilling these three equations graphically on Fig. 115 we get the three e.m.f.'s E_1, E_2, and E_3 as shown (Fig. 115).

In the triangle oab $oa = ab$ $(e_3 = e_2)$

therefore $\angle boa = \angle oba$

but $\angle oab = 120°$

therefore $\angle boa = 30°$

Drop a perpendicular ax from a on ob

then $$E_1 = 2\,ox = 2\,e_3 \cos 30° = 2\,e_3 \frac{\sqrt{3}}{2} = \sqrt{3}\,e_3$$

In all three-phase generators $e_1 = e_2 = e_3$

therefore $E_1 = \sqrt{3}\,e$

likewise $E_2 = \sqrt{3}\,e$ and $E_3 = \sqrt{3}\,e$ or $E_1 = E_2 = E_3$

or in a Y-connected system any line voltage $= \sqrt{3}$ times any phase voltage.

51. Three-phase △- (Delta) (sometimes called mesh) connected System.—Notice Fig. 116 carefully and you will see that the three phases a, b, and c are in exactly the same position as in Fig. 114, and of course are 120° apart.

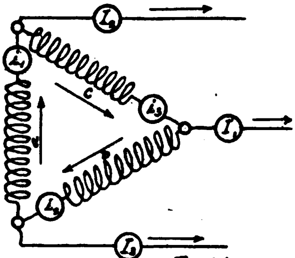


FIG 116

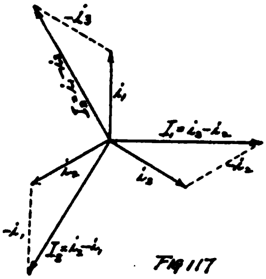


FIG 117

When connected as shown in Fig. 116, the generator is said to be connected △ (delta).

Since each line is in parallel with each phase, the phase e.m.f.'s equal the line e.m.f.'s;

that is $e = E$

It therefore remains to establish the relations between the phase currents and the line currents, or between i and I.

For this purpose assume a symmetrical choice of signs; that is, assume the phase currents are positive when flowing clockwise as shown by the arrows (Fig. 116). Further assume that the line currents are considered positive when flowing away from the generator as shown by the arrows (Fig. 116).

Evidently the current in the upper main (Fig. 116) is made up of the two-phase currents i_1 and i_3, and therefore called I_2.

Similarly I_1 is made up of i_2 and i_3

and I_3 is made up of i_1 and i_2

∴ Fig. 116 is symmetrically lettered.

Now, i_1, and i_2, and i_3 are 120° apart, as shown (Fig. 117).

From the choice of signs given in Fig. 116:—

$$I_1 = i_3 - i_2$$
$$I_2 = i_1 - i_3$$
and
$$I_3 = i_2 - i_1$$

Fulfilling these three equations graphically we get the three currents, I_1, I_2, and I_3, as shown in Fig. 117.

Fig. 117 is similar to Fig. 115, and in the same way it can be proved that:—

$$I_3 = \sqrt{3}\, i_3 \qquad \text{or since } I_1 = I_2 = I_3$$

in a △-connected system any line current $= \sqrt{3}$ times any phase phase current.

52. Summary.—In any Y-connected system:—

(1) The line current equals the phase current $I = i$

(2) The line e.m.f. equals $\sqrt{3}$ times the phase e.m.f. $E = \sqrt{3}\, e$ (52)

In any △-connected system:—

(1) The line e.m.f. equals the phase e.m.f. $E = e$

(2) The line current equals $\sqrt{3}$ times the phase current $I = \sqrt{3}\, i$ (53)

53. Power Expressions for Y-connected Systems.—

$$P_{(\text{per phase})} = e\, i \cos \theta$$

where θ is the angle between e and i.

$$P_{(\text{total})} = 3\, P_{(\text{per phase})} = 3\, e\, i \cos \theta \tag{a}$$

In a Y system $e = \frac{E}{\sqrt{3}}$ and $i = I$

Substituting in equation (a) we get:—

$$P_{(\text{total})} = 3 \frac{E}{\sqrt{3}} I \cos \theta = \sqrt{3}\, E\, I \cos \theta \tag{54}$$

54. Power Expressions for △-connected Systems.—

$$P_{(\text{power phase})} = e\, i \cos \theta$$

where θ is the angle between e and i

$$P_{(\text{total})} = 3\, P_{(\text{per phase})} = 3\, e\, i \cos \theta \tag{b}$$

In a △ system $e = E$ and $i = \frac{I}{\sqrt{3}}$

Substituting in equation (b) we get:—

$$P_{(\text{total})} = 3\ E \frac{I}{\sqrt{3}} \cos\theta = \sqrt{3}\ E\ I \cos\theta \quad \text{(see equation 54)}$$

or the total power in any three-phase Y or △ system equals the line voltage times the line current times the cosine of the angle between the phase voltage and the phase current.

$$P = \sqrt{3}\ E\ I \cos\theta \qquad \text{(see equation 54)}$$

55. The Two-wattmeter Method of Measuring Power in a Three-phase System.—One of the most convenient ways of measuring the total power delivered to a three-phase receiving circuit

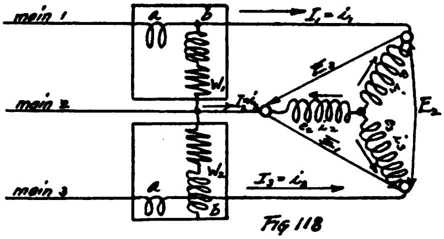


Fig 118

is to use two wattmeters connected as shown in Fig. 118. The method is true for either Y or △ circuits. We have assumed a perfectly symmetrical choice of signs as shown in Fig. 118.

Evidently $W_1 = \text{average } E_3\, i_1$,

$W_2 = \text{average } E_1\, i_3$

or $W_1 + W_2 = \text{average } (E_3\, i_1 + E_1\, i_3)$ (55)

but $E_3 = e_2 - e_1$ (a)

and $E_1 = e_3 - e_2$ (b)

therefore substituting in equation (55):—

we get $W_1 + W_2 = \text{average } (e_2\, i_1 - e_1\, i_1 + e_3\, i_3 - e_2\, i_3)$ (56)

but $P_{(\text{total})} = \text{average } (e_1\, i_1 + e_2\, i_2 + e_3\, i_3)$ (57)

Equation 56 does not contain i_2 (as there is no current coil in main 2, Fig. 118). Therefore we will eliminate i_2 from equation (57):—

Evidently $i_1 + i_2 + i_3 = 0$

therefore $i_2 = -i_1 - i_3$

substituting this value of i_2 in equation 57 we get:—

$$P_{(\text{total})} = \text{average } (e_1\, i_1 - e_2\, i_1 - e_2\, i_3 + e_3\, i_3) \qquad (58)$$

Equating equations (56) and (58):—

we get $\text{average}\,(e_2\,i_1 - e_1\,i_1 + e_3\,i_3 - e_2\,i_3) = \text{average}\,(e_1\,i_1 - e_2\,i_1 - e_2\,i_3 + e_3\,i_3)$
or $e_2\,i_1 - e_1\,i_1 = e_1\,i_1 - e_2\,i_1$
or $(e_2 - e_1)\,i_1 = (e_1 - e_2)\,i_1$
but $e_2 - e_1 = E_3$ equation (a)
and $e_1 - e_2 = -E_3$ equation (a)
therefore $E_3\,i_1 = -E_3\,i_1$

So we see that the two wattmeters measure all the power. From this final expression it is seen that one wattmeter may read backward. However, by reversing the leads of its potential coil both meters will read forward. If the power factor is less than .5, it can easily be shown that one meter will read backward. To determine whether the readings shall be added or subtracted proceed as follows:—

Subtract the two readings if on substituting one meter for the other (the connection to main 2, Fig. 118, being unaltered) the deflection reverses; otherwise the two readings are to be added. The current coil connections between a and b must be made to the line in each case as shown.

An easier but not as practical a method is to connect the two meters so they will read positive on a lamp load (unity p.f.), and then on substituting the actual load if either meter reads backward its pressure coil lead must be reversed and the two readings subtracted. (See Sheldon, Mason, and Hausmann, "Alternating Current Machines," pp. 108 to 113, for other methods of measuring power.)

In getting the average $e_1\,i_1$, $e_2\,i_1$, etc., the algebraic sum is meant, since the average values of $e_1\,i_1$, $e_2\,i_1$, etc., may be of opposite sign.

The sum of the two-wattmeter readings equals the total power, and is independent of whether the load is balanced or not, and is also independent of the wave shape of current and e.m.f., provided the wave shape of the current in the potential coil is the same as the wave shape of the potential across it. This condition is only approximately fulfilled when the potential circuit of the meter contains iron.

Three-phase generators and motors are generally connected Y in order to obtain as high a line voltage as possible for economic transmission with a lower voltage on the windings of the machines. Transformers are generally connected △ (see art. 112), and lamps and rotary converters are connected △ from the nature of the case. (See art. 145.)

56. The Different Systems Compared as to Relative Copper Economy.—The fairest comparison can be made by making the following assumptions in the two examples below:—

1. The same line voltage in each case (E = 1000).
2. The same power factor in each case (p.f. = unity, lamp load).
3. The same length of transmission line.
4. The same total power delivered in each case (10,000 watts).
5. The same loss of power in the mains in each case ($I^2R = 200$ watts).

Example 1:—Single-phase versus two-phase four-wire.

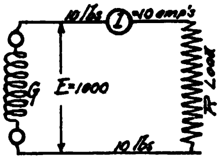


FIG 119

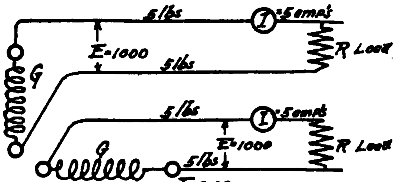


FIG 120

Single-phase	Two-phase four-wire
$I^2 R = 200$ watts	$I^2 R = 200$ watts
	$I^2 R_{(\text{per phase})} = 100$ watts
$R = \frac{200}{(10)^2} = 2$ ohms (total)	$R = \frac{100}{(5)^2} = 4$ ohms (total)
or R = 1 ohm for each wire	or R = 2 ohms for each wire
Assume a wire of 1 ohm weighs 10 pounds;	then a wire of 2 ohms same length weighs 5 pounds
or total copper = 20 pounds	or total copper = 20 pounds

Example 2:—Two-phase four-wire versus three-phase Y or △.

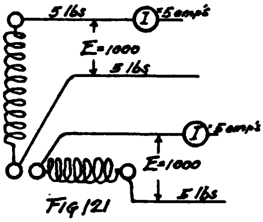


FIG 121

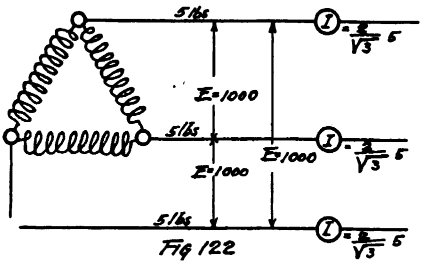


FIG 122

Total weight of copper = 20 pounds.	Total weight of copper = 15 pounds.

Let R_2 = resistance of each wire of the two-phase line.

Let R_3 = resistance of each wire of the three-phase line.

Let I_2 = current in each wire of the two-phase line.

Let I_3 = current in each wire of the three-phase line.

Then since P is the same in each case and p.f. is unity—

we have $$2\,E\,I_2 = \sqrt{3}\,E\,I_3$$

or $$I_2 = \frac{\sqrt{3}}{2} I_3$$

Now, $4\,R_2\,I_2^2$ is total power lost in two-phase line, and $3\,R_3\,I_3^2$ is total power lost in three-phase line, and by the conditions of the problem these two are equal—

or $$4\,R_2\,I_2^2 = 3\,R_3\,I_3^2 \qquad \text{(a)}$$

substituting $$I_2 = \frac{\sqrt{3}}{2} I_3 \text{ in equation (a)}$$

we get $$4\,R_2\,\frac{3}{4}\,I_3^2 = 3\,R_3\,I_3^2$$

or $$R_2 = R_3$$

But since there are four wires in the two-phase system and but three wires in the three-phase system, it follows that the weight of copper in the two-phase line is $\frac{4}{3}$ times as great as in the three-phase system.

This result is independent of the power factor provided the power factor in the two-phase system is the same as in the three-phase system.

57. Summary.—The advantages of the three-phase system over the single-phase system are:—

1. The saving in the amount of copper (25%, see problem 128) as compared with the single-phase system for the same voltage between wires.

2. The more satisfactory operation of three-phase motors compared with single-phase ones.

3. The smaller cost of three-phase generators and motors of the same output and voltage.

4. Better voltage regulation of three-phase generators.

The advantages of the three-phase system over the two-phase four-wire system are:—

1. The saving in the amount of copper ($33\frac{1}{3}$%, see example 2).

2. The use of three wires instead of four.

The advantages of the three-phase system over the two-phase three-wire system are:—

1. While the two-phase three-wire takes less copper by the

ratio of 2.914 to 3 (problem 127), its highest voltage across the outside wires is $\sqrt{2}$ E, compared to E in the three-phase system.

2. The voltage drop in the middle main may cause the voltage across the two outside mains to become unequal to each other and more or less than 90° apart, which is undesirable in the operation of lamps and motors.

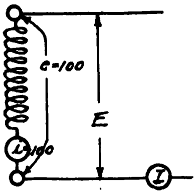
3. Unless transformers are used a two-phase four-ring converter cannot be connected with two-phase three-wire mains. (See problem 190.)

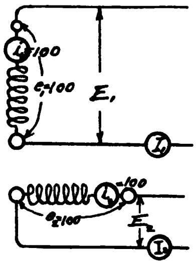
PROBLEMS.

(*Note:*—Make a clock diagram for each problem.)

99. On a balanced two-phase three-wire system the current in each phase is equal to 200 amperes; what is the current in the common return wire? *Ans.*:—282.8 amps.

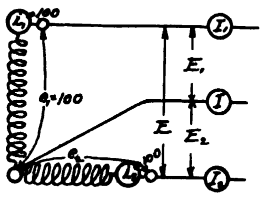
100. The e.m.f. of each phase of problem 99 is 2200 volts; what is the e.m.f. across the outside wires? *Ans.*:—3110.8 volts.

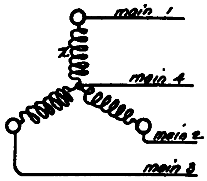




101. In the single-phase circuit shown find: (*a*) E, (*b*) I, (*c*) power in terms of e and i, and (*d*) power in terms of E and I. *Ans.*:—Power = 10,000 watts.

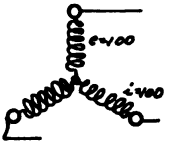
102. In the 2-phase 4-wire circuit shown find: (*a*) E,, (*b*) I,, (*c*) E_2, (*d*) I_2, (*e*) power in each phase, and (*f*) total power. *Ans.*:—(*e*) 10,000 watts; (*f*) 20,000 watts.



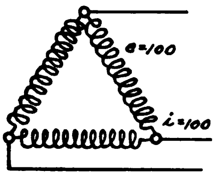


103. In the two-phase three-wire circuit shown find E,, E_2, E, I,, I_2, and I. Find total power output in terms of e, and i,, in terms of E, and I,, in terms of E and I,, in terms of E_2 and I, and in terms of E and I. What is power delivered by each phase? *Ans.*:—$P_{\text{(per phase)}}$ = 10,000 watts; $P_{\text{(total)}}$ = 20,000 watts.

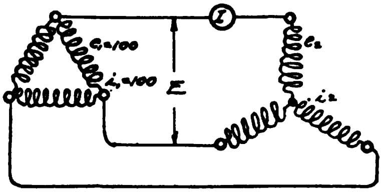
104. A three-phase generator is provided with four collector rings and mains as shown at bottom of page 81. Three similar receiving circuits are connected as follows: one from main 1 to main 4, one from main 2 to main 4, and one from main 3 to main 4. Each receiving circuit takes 500 amperes. With the armature windings properly connected to the collector rings, what is the current in main 4? What is the current in main 4 when one armature winding has its connections reversed? Make a diagram showing the phase relations between the currents in mains 1, 2, 3, and 4; also when one of the armature windings x is reversed. In the diagrams consider currents away from ring 4 as positive. *Ans.*:—(*a*) 0 amperes; (*b*) 1000 amperes.



(a)



(b)



(c)

105. The electromotive force developed in each winding of a three-phase generator is 500 volts. What is the electromotive force developed by the three windings connected in series? *Ans.*:—1000 volts or o volts.

106. Find line voltage and current (I) in the circuit shown (Fig. a). Find power delivered. *Ans.*:—P = 30,000 watts.

107. Find line voltage (E) and line current (I) in the circuit shown (Fig. b). Find power delivered. *Ans.*:—P = 30,000 watts.

108. Find E, I, e_2, i_2, and assuming perfect efficiency find power delivered by the generator to the motor in the circuit shown (Fig. c). *Ans.*:—$e_2 = 57.8$, $i_2 = 171$, P = 30,000 watts.

109. Three similar receiving circuits are △-connected to three-phase mains, the current in each pair of mains being 875 amperes (I). The total power delivered to the three circuits is 300 kilowatts and the power factor of each circuit is 0.9. What is the voltage across each receiving circuit and across each main? *Ans.*:—220 volts.

110. Three similar receiving circuits are Y-connected to the three-phase mains of problem 109, the total power delivered is 300 kilowatts and the power factor of each circuit is 0.9. What is the current in each circuit and in each main, and what is the voltage between the terminals of each receiving circuit? *Ans.*:—875 amps., 875 amps., 127 volts.

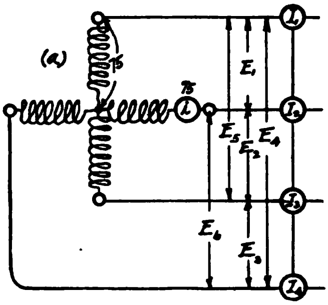
111. A given three-phase generator has its armature connected to six rings so that it can be connected either △ or Y. Each of the three similar receiving circuits has a resistance of 6 ohms and a reactance of 8 ohms. The e.m.f. generated in each armature winding is 400 volts. If the generator is △-connected to the line and the receiving circuit Y-connected to the line, find:—
    (*a*) the current in each receiving circuit.
    (*b*) the current in each line.
    (*c*) the current in each armature winding.
    (*d*) the total power delivered.
Ans.:—(*a*) 23.1 amps.; (*b*) 23.1 amps.; (*c*) 13⅓ amps.; (*d*) 4800 watts.

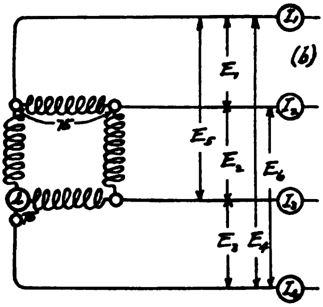
112. Work out the results called for in problem 111, assuming the generator Y-connected to the line and the receiving circuit △-connected to the line. *Ans.*:—(*a*) 20 amps.; (*b*) 34.6 amps.; (*c*) 34.6 amps.; (*d*) 4800 watts.

113. Determine the voltage across the outside terminals of a two-phase three-

wire 100-volt generator when the windings on its armature are set 85° apart instead of 90° apart.

114. Find the magnitude and phases of the various voltages and currents across and in the four-line wires of the four-phase systems shown, if the voltage generated in each armature winding is 75 volts and the current in each armature winding is 75 amperes.





115(*a*). In the two-wattmeter method of measuring power in a three-phase system and with a lamp load the following readings were taken:—

$W_1 = 2000$ watts
$W_2 = 2000$ watts

with a motor load the two readings were:—

$W_1 = 2000$ watts
$W_2 = 0$ watts

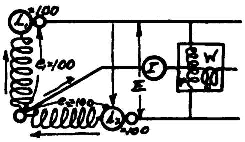
at the instant of throwing the motor on the line the two readings were:—

$W_1 = 2750$ watts
$W_2 = 1250$ watts (after having its pressure lead reversed)

What is the total power delivered in each case?
What is the power factor in each case?

115(*b*). Prove that 2 wattmeters will measure all the power in a three-phase Δ-connected system if the meters are connected as shown in Fig. 118.

116. With the choice of signs shown below, show by a clock diagram that E and I are 90° apart. What will the wattmeter read?



117. If in problem 116 the phase e.m.f.'s and phase currents are 30° apart (current lagging), what will be the angle between E and I?
(*a*) What will the wattmeter read? Will it read backward?
(*b*) What will the wattmeter read if the current leads by 30°?
Ans.:—(*a*) 10,000 watts negative.
(*b*) 10,000 watts positive.

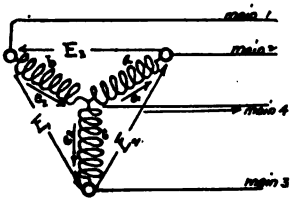
118. If in the figure of problem 116—

$e_1 = 100$ $i_1 = 50$
$e_2 = 100$ $i_2 = 86.6$ and unity p.f. in each phase

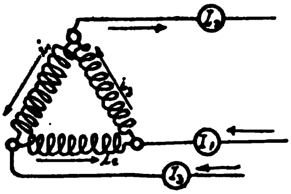
(*a*) What will the wattmeter read?

(*b*) With all other conditions unchanged, what will the wattmeter read if—
$i_1 = 86.6$
$i_2 = 50$
Ans.:—(*a*) 3745 watts negative.
(*b*) 3745 watts positive.

119. Solve problem 118(*a*) assuming:—
(*a*) $\left.\begin{array}{l} i_1 \text{ lags behind } e_1 \\ i_2 \text{ lags behind } e_2 \end{array}\right\}$ by 30°
(*b*) $\left.\begin{array}{l} i_1 \text{ leads } e_1 \\ i_2 \text{ leads } e_2 \end{array}\right\}$ by 30°
Ans.:—(*a*) 10,000 watts negative.
(*b*) 3745 watts positive.



(a)



(b)

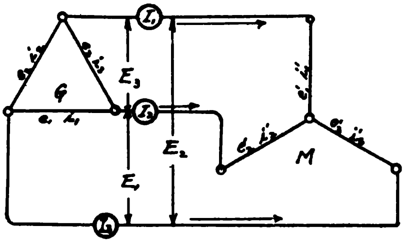
120. In the balanced three-phase four-wire system shown and with the choice of signs shown, prove that the current in main 4 is zero.

121. With the choice of signs indicated in figure a show that $E = \sqrt{3}e$ in a three-phase three-wire Y-connected system.

122. With the choice of signs shown in figure b prove that $I = \sqrt{3}i$.

123. Assuming unity power factor in each phase show that E and I are 30° apart in:—
(*a*) △-connected system.
(*b*) Y-connected system.

124. Assume a △-connected generator, unity p.f., connected to a Y-connected motor, unity p.f., as shown. Make a complete clock diagram for G, line, and M, and get all the phase relations. What is the phase relation between:—
(*a*) e_1 and e'_1; (*b*) e_2 and e'_2; (*c*) e_3 and e'_3.
Ans.:— $\left\{\begin{array}{l} (a)\ e'_1 \text{ is } 90^\circ \text{ ahead of } e_1. \\ (b)\ e'_2 \text{ is } 90^\circ \text{ ahead of } e_2. \\ (c)\ e'_3 \text{ is } 90^\circ \text{ ahead of } e_3. \end{array}\right.$



125. Make a clock diagram for a three-phase Y-connected generator connected to incandescent lamps connected △ across the three lines. If one phase of the generator should burn out, make a clock diagram for the single-

phase circuit which remains. Are e and i still in phase? *Ans.:*—No; 30° apart.

126. Which system takes the more copper and how much more—three-phase six-wire or three-phase three-wire?
Assumptions:—
    1. The same line voltage in each case ($E = 1000$).
    2. The same p. factor (p.f. = unity).
    3. The same length of transmission line.
    4. The same total power delivered in each case ($P = 1000$ watts).
    5. The same loss of power in the mains ($I^2R = 200$ watts).

    Ans.:—Six-wire takes 20% more copper than three-wire.

127. Compare two-phase three-wire system with three-phase three-wire system under the assumptions given in problem 126 as to copper economy. *Ans.:*—The three-phase system requires less copper in the ratio of 2.914 to 3 (2.9% less).

128. Compare a single-phase system with a three-phase three-wire system under above assumptions given in problem 126 as to copper economy. *Ans.:*—Single-phase system takes 25% more copper than three-phase system.

129. Compare the three-phase three-wire system with the two-phase four-wire system as to the amount of copper necessary in each, under the assumptions given in problem 126, only change assumption 5 to read thus:—
    5. The same IR drop in the mains in each case.

    Ans.:—The two-phase line has $\frac{4}{3}$ times as much copper as the three-phase line. ($33\frac{1}{3}$% more).

CHAPTER VIII.

GENERATORS.

A generator is a machine which converts mechanical energy into electrical energy. There are two general types: First, the D.C. generator, which generates A.C. in its armature and rectifies it into D.C. by means of a commutator; second, the A.C. generator, which generates A.C. in its armature and delivers that A.C. directly to the line as A.C. by connecting the ends of the armature windings to slip rings, which are ordinary copper rings with brushes, connected with the external circuit, bearing on them. Both machines have D.C. field excitation, and the only radical difference is the fact that the A.C. generator does not need a commutator.

58. D.C. Generator used as an A.C. Generator.—Let the circle (Fig. 123) represent the drum armature of a two-pole D.C. generator containing, say, 300 conductors. Suppose this closed coil winding to have the two points, A and B, soldered to two collector rings. When the two connections A and B are in the positions shown in Fig. 123, the A.C. e.m.f. induced will be a maximum, since all the conductors (being in series) between A and B have their e.m.f.'s adding to each other (all those on the left being under a N-pole and those on the right being under a S-pole).

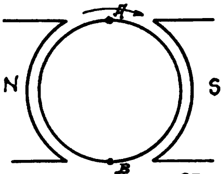


FIG 123

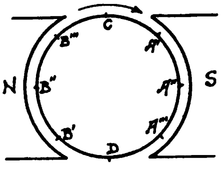


FIG 124

In Fig. 124 the armature has rotated clockwise so that the ring connections A and B are now in the positions A′ and B′. Now all the coils between A′ and B′ do not have their e.m.f.'s adding, as most of those on the left are under a N-pole and some under a S-pole. In fact, the e.m.f. between rings now is B′C−A′C, so that the e.m.f. is less than it was in Fig. 123. Clearly when the ring connections have reached the horizontal positions B″A″, the e.m.f. induced is B″C − A″C = zero.

The sine wave of e.m.f. which may thus be obtained is shown in Fig. 125, on which the positions of A and B of Figs. 123 and 124 are shown.

In Fig. 126(*a*) a phase connected to the points A B has a maximum e.m.f. in the position shown, while a phase connected to the points C D has zero e.m.f. at the same instant; these two e.m.f.'s are 90° apart. Hence, connecting one external circuit to A B and the other to C D, we have a two-phase system, and this machine becomes a two-phase generator.

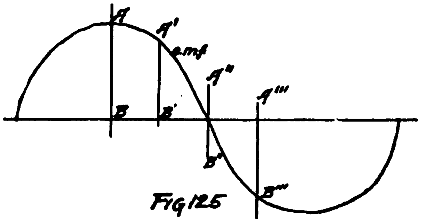


Fig 125

Again, by taking three points, A, B, and C, Fig. 126(*b*), which are clearly 120° apart, and connecting them to three slip rings, we obtain a three-phase generator. Such a mode of construction is used in practice when it is desired to take off both D.C. and A.C., in which case the machine is called a double-current generator, or when a transformation from A.C. to D.C. or vice versa is desired, it is called a rotary converter.

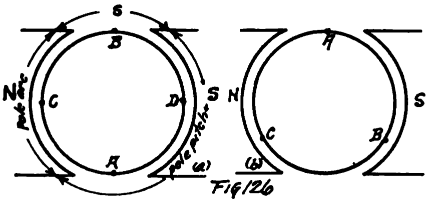


Fig 126

59. Differences in Construction Between A.C. and D.C. Generators.—If the machine is only required to generate A.C., the above arrangement can be considerably improved. In Figs. 123 and 124 the only positions of the armature when the e.m.f. in some of the coils is not opposing the e.m.f. in others is in the vertical position, Fig. 123. In the position of A′ B′, Fig. 124, the e.m.f.'s in the coils C A′ are opposing the e.m.f.'s in the coils B′ C. This opposition of e.m.f.'s is called *differential* action. In the position A″ B″, Fig. 124, where the resultant e.m.f. is zero, the differential action is 100%.

Now, this differential action is objectionable for two reasons:—First, it unnecessarily adds to the resistance and reactance of the armature; and, second, it reduces the e.m.f.

In order to eliminate differential action it is necessary to arrange the windings so that all conductors forming a group of the winding between two contact rings shall always be moving across a field of the same polarity. To attain this result the width of a band of conductors should not exceed the width of the interpolar space (s in Fig. 126a).

In D.C. generators the ratio of pole arc to pole pitch is generally about .7 (see Fig. 126-a); if we used the same ratio of pole arc to pole pitch (.7) in A.C. generators, in single-phase machines, we could only use 30° of the surface of the armature. In order to increase the width of the winding without introducing any appreciable differential action, it is customary to make the pole arc one-half the pole pitch. This reduction of pole arc is one of the constructional differences between single-phase A.C. and D.C. generators.

With this arrangement only one-half the available space on the armature is used, while on a D.C. generator the entire armature is uniformly wound. If the A.C. machine is to be two-phase, the intervening spaces are occupied by the second phase, so in a two-phase generator the entire armature space is utilized.

In a three-phase machine the pole pitch has to be divided into three parts, and the width of each of these represents the width of a band of conductors in any one phase. Here again the entire available space on the armature is utilized. Since in the three-phase generator the width of a band belonging to any one phase does not exceed one-third the pole pitch, it follows that the pole arc may be as large as two-thirds of the pole pitch, and this is generally the case in three-phase generators.

It is common practice in single-phase service to supply a generator with a three-phase winding and use any two of the three leads; the single-phase rating is about 70% of the standard three-phase rating.

A.C. generators generally have more poles than D.C. machines in order to obtain the present commercial frequencies. This is not true of turbine-driven units. *Example:*—A 25-cycle 75 r.p.m. generator must have forty poles.

60. Rating of A.C. Generators.—The size of an A.C. generator is determined by the current and the e.m.f. required, and is expressed in kilo-volt amperes (abbreviated to K.V.A.). *Example:*—A generator has the following information on the maker's name-plate—"2000 K.V.A. 10,000 volts." This rating is always made at unity power factor.

$$\text{I (at unity p.f.)} = \frac{2,000,000}{10,000} = 200 \text{ amperes}$$

which is the full load current of the machine.

At .8 p.f. the full
load output will be reduced to $10,000 \times 200 \times .8 = 1600$ K.W.
at unity
p.f. the full load output will be $10,000 \times 200 \times 1 = 2000$ K.W.

In determining the size of a prime move to drive the generator it is very important to know what the full load power factor is likely to be, as the size of the prime mover is determined by the energy required expressed in K.W. or H.P.

61. General Performance.—*Sample:*—This generator will deliver its full rated K.V.A. for twenty-four hours with a rise in temperature not exceeding 40° C. and will carry 25% increase of current for twenty-four hours with a temperature rise not exceeding 50° C., and will carry a 50% increase of current for one hour with a temperature rise not exceeding 60° C.—(Westinghouse Elec. and Mfg. Co.).

A better guarantee than this may be obtained by paying more for the generator.

62. Types of A.C. Generators.—There are three distinct types of A.C. generators:

First: The ordinary revolving armature stationary field type. In this respect the A.C. and D.C. generators are alike.

Second: Stationary armature, revolving field. This type is the most common of all, and has several advantages over the first type.

1. It avoids the collection of high-tension currents through slip rings and brushes, since the armature connections are sta-

tionary and only D.C. at low e.m.f. (generally not over 125 volts) need be taken through rings to the field coils.

2. It allows more room for insulation of armature which would be much sooner damaged if the armature revolved. Revolving field machines have been built to generate 25,000 volts, while the limit in revolving armature types is about 5,000 volts, due to the inability to insulate rapidly moving wires effectively.

3. In the case of polyphase machines the revolving field type avoids the use of more than two slip rings. (For constructional features and illustrations see G. E. or Westinghouse bulletins and the A.C. generators in the laboratory.)

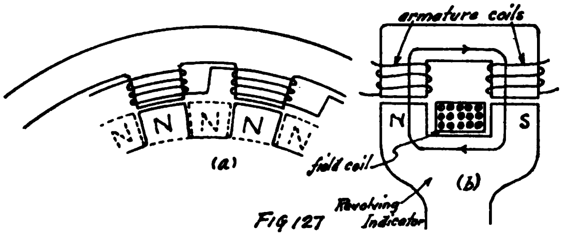


FIG 127

Third: The inductor type generator. This type has neither slip rings nor revolving armature nor revolving field, but has a rotating magnetic circuit consisting of a mass of iron called an inductor.

Fig. 127 shows two views of the inductor type generator. There is only one field coil, which is stationary, and this field excited by D.C. produces flux through the revolving inductor as shown (assuming current to be flowing away from the reader in the field coil). When the pole pieces are directly under the armature projections (Fig. 127-a), the magnetic reluctance across the air gap is a minimum, and therefore the flux is a maximum and the e.m.f. is zero. When the poles have passed to a position midway between the armature projections, as shown by the dotted lines (Fig. 127-a), the reluctance is a maximum and the flux is a minimum, and the e.m.f. is a maximum (maximum rate of change). As the inductors revolve the linked flux changes from a maximum to a minimum, but does not change in direction.

The poles are staggered with respect to each other, like the spokes of a wheel (Fig. 128). As in all A.C. generators the polar projections on inductor and armature are laminated to prevent eddy currents.

Advantages:—This machine has no moving wire, and therefore no insulation to chafe and no collecting devices with their attendant brush friction. It is therefore claimed that these machines have better facilities for insulation. They may be wound for single or polyphase (more than one phase) currents.

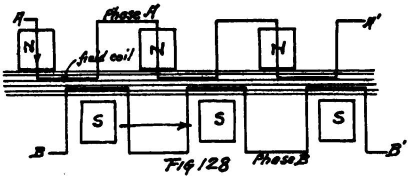


Fig 128

64. **Armature Windings.**—There are two general types of armature windings—concentrated and distributed armature windings.

A *concentrated* winding is one in which all the wires on the armature are bunched together in p slots, where p is the number of field poles.

Fig. 129 represents a concentrated winding for a four-wire single-phase generator, one conductor per slot. The armature is moving as indicated by the arrow. (The 1000-volt La Roche A.C. generator in the dynamo laboratory, Drexel Institute, has a concentrated armature winding.)

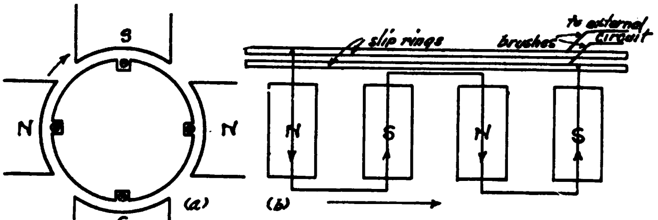


Fig 129

A *distributed* winding is one in which all the wires on the armature are placed in more than p slots, as in all the examples which follow.

A concentrated winding is generally placed in rather large slots, each containing many wires. In a concentrated winding the e.m.f. rises very suddenly as the slots pass into the polar region, and falls suddenly as the slots pass out of the polar region, while in a distributed winding the e.m.f. rises and falls more gradually.

65. The Six Slots per Pole Winding.—The advantages of choosing six slots per pole lies in the possibility of standardizing core plates so that the same core may be used for single-, two-phase, and three-phase machines. While six slots per pole are generally used three slots per pole and twelve slots per pole are frequently found.

66. General Rules to Follow.—*Rule 1.*—In order that the A.C. wave of e.m.f. may be unbroken the coils of each phase must occur every two poles (360°) or every twelve slots.

Rule 2.—In order that the e.m.f.'s induced in the two sides of a coil will add to each other it is necessary that a coil span from a north to a south pole or the average pitch for all coils should be six slots.

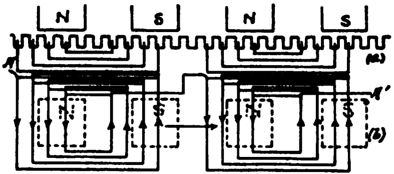


Fig 130

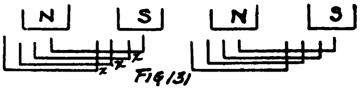


Fig 131

67. The Single-phase Winding.—Fig. 130 represents a four-pole six slots per pole single-phase winding. Two-thirds of the slots on the armature are occupied. Since a group of coils must occur every two poles (rule 2) or every twelve slots, and only two-thirds or eight are occupied, we can have four coils every pair of poles.

If the average pitch is to be 6, the four pitches must be 3, 5, 7, 9 (Fig. 130-a). The reason each coil does not have a pitch of 6 (Fig. 131) is because of the mechanical impossibility of bringing out the coils and bending them over each other at the points xxx, especially on high-current machines.

If the entire armature were occupied, there would be six coils per pair of poles with pitches 1, 3, 5, 7, 9, 11 (average, 6), but the inner coil (pitch 1) and the outer coil (pitch 11) are so far from the proper pitch (pitch 6) that most of the time both sides of these two coils are under poles of the same polarity, and therefore have practically no useful e.m.f. generated in them (differential action). Since they would lower the efficiency by increasing the I^2R losses, they are not used. Sometimes the unused slots are occupied by a small second phase, so that the generator can supply a two-phase motor if need be in connection with its single-phase load.

68. The Two-phase Winding.—Fig. 132 represents a four-pole six slots per pole, 2-phase winding.

Each phase must occur every two poles (rule 1) or every twelve slots; therefore each phase can occupy six slots per pair of poles. We can get three coils in six slots; therefore there will be three coils per phase per pair of poles with the three pitches 4, 6, and 8 (average 6). It will be noticed that phase B is 90° from phase A (Fig. 132-a).

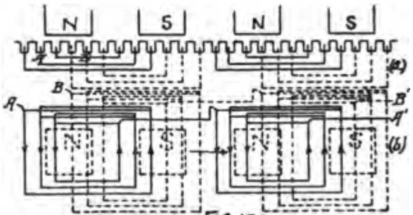


Fig 132

69. The Three-phase Winding.—Fig. 133 represents a four-pole, six slots per pole, three-phase winding.

Each phase must occur every two poles (rule 1) or every twelve slots. Since there are three phases, each phase can occupy four slots per pair of poles.

We can get two coils in four slots; therefore there will be two coils per phase per pair of poles with the two pitches 5 and 7 (average 6). It will be noticed that the right-hand side of phase B is four slots, or 120°, from the right-hand side of phase A, and in the same manner the right-hand side of phase C is 120° from the right-hand side of phase B.

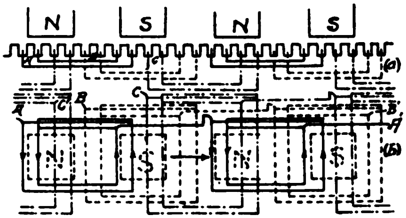


Fig 133

If the armature (Fig. 133-a) move from left to right, you will notice that the phases occur in the order C. B. A. If the armature (Fig. 133–a) move from right to left, you will notice that the phases occur in the order A. B. C. Phases A and C have been interchanged; thus we see that a change in the reversal of direction of a three-phase generator is equivalent to interchanging the connections of two of its terminals as far as the external circuit is concerned.

70. Summary.—

DISTRIBUTED WINDINGS

	3 Slots per Pole			6 Slots per Pole			12 Slots per Pole		
	One-phase	Two-phase	Three-phase	One-phase	Two-phase	Three-phase	One-phase	Two-phase	Three-phase
Average pitch	3	3	3	6	6	6	12	12	12
Coils per phase per pair of poles......	2	—	1	4	3	2	8	6	4
Pitches......	2, 4	—	3	3, 5, 7, 9	4, 6, 8	5, 7	5, 7, 9, 11, 13, 15, 17, 19	7, 9, 11, 13, 15, 17	9, 11, 13, 15

From the manufacturing point of view armature coils may be divided into two classes—former-wound and hand-wound.

Former-wound coils require entirely open slots, whereas hand-wound coils may be placed in closed or semi-closed slots. Former-wound coils are cheaper and may be more easily replaced. On the other hand, in fitting former-wound coils into position there is some risk of damaging the insulation of the coil or slots. The use of open slots results in an irregular wave shape unless some such expedient as skewing the pole pieces is adopted (used by the Oerlikon Co.) (Fig. 134), and this necessitates, as a rule, the use of laminated pole shoes in order to prevent excessive eddy current losses. Owing to the fact that hand-wound coils admit of more effective insulation (the insulating lining of the slot in this case consisting of a seamless tube, generally some form of mica); this type would appear to be preferable in the case of generators of very high voltages. Former-wound coils are the more generally used.

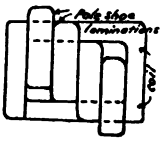


FIG. 134

(For details of armature and field construction see Westinghouse circulars Nos. 1133, 1190, etc.)

71. The Fundamental Equation of an A.C. Generator.—It is

desired to derive an expression for the e.m.f. generated in the armature of an A.C. generator.

Let ϕ = flux coming from or entering each field pole; then—

2ϕ = flux cut in one cycle by each conductor on the armature.

if f = frequency in cycles per second, then—

$2\phi f$ = flux cut in one second by each conductor on the armature.

If C = number of conductors connected in series in each phase, then the volts generated per phase—

$$E = \frac{2\phi f C}{10^8} \tag{59}$$

Since each conductor on the armature passes between poles as well as under poles, the E in equation (59) is $E_{(\text{average})}$.

Assuming a sine wave $\quad E_{(\text{r.m.s.})} = E_{(\text{average})} \times 1.11$

or $\quad E_{(\text{r.m.s.})} = \dfrac{2.22\,\phi f C}{10^8}$ (compare with equation 15) (60)

Equation (60) is true only for a concentrated winding, and under the assumption that the e.m.f wave is a sine wave.

With a distributed winding the e.m.f. is reduced because all the conductors in a given phase are not in the thickest field at the same instant.

For example: Consider a three-phase six slots per pole generator. In Fig. 133-a it is seen that the e.m.f.'s generated in the adjacent coils in phase A are one slot or 30° electrical degrees out of phase with each other, and while all the coils of the phase are connected in series, the total voltage is slightly less than it would be if the two sets of coils were placed in the same slots. (Concentrated winding.)

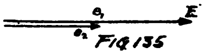


FIG 135

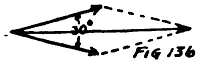


FIG 136

Let e_1, Fig. 135, represent the e.m.f. generated in the group of coils with a pitch of 7; then e_2 would be the e.m.f. generated in the group of coils with a pitch of 5, and if they were in the same slots with the first group, the total e.m.f. of that phase would be:—

$$E = 2e_1 = 2e_2$$

Since e_1 and e_2 are in adjacent slots, or 30° electrical degrees apart, the total e.m.f. E will be $2e_1 \cos 15°$ (Fig. 136) or less than

2 e, by cos 15°. Therefore a three-phase six slots per pole generator (equation 60) should be modified to read:—

$$E = 2.22\ \phi f C \cos 15°$$

and similarly for other types of windings.

As seen from problem 39, the form factor is not 1.11 for any except a sine wave, so that the shape of the e.m.f. wave will change the constant 2.22 to some other value, and of course any increase in the form factor will increase the e.m.f.

The wave shape of e.m.f. induced in a single conductor on the armature surface is identical with that of the distribution of magnetic flux at the armature surface; this will be discussed more fully under armature reactions.

(For methods of tracing pressure and current curves, etc., see D. C. and J. P. Jackson, "Alternating Currents," pp. 289–306.)

In general equation

$$E = 2.22\ \phi f C \cos 15° \text{ should take the form}$$
$$E = k\phi f C \qquad (61)$$

where k takes care of 2.22×cos 15°. Since 2.22 and cos 15° will vary for different types of generators, k has a different value in each case.

72. Armature Reactions.—As we have learned in direct-current machines, armature reactions are the reactions of the flux produced by current flowing through the armature, on the field flux; and since the amount of armature flux depends on the number of armature ampere terms ($\phi = \frac{4}{10}\pi\frac{AT}{R}$, where A = amperes, T = turns of wire, and R = reluctance of the magnetic path) on the armature, it is easy to see that the armature reactions increase in direct proportion to the armature current, since the number of turns on the armature cannot change.

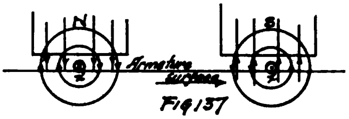


Fig 137

(*a*) *Pure Resistance Load.*—In Figure 137 consider a two-pole generator with armature moving from left to right as indicated by the arrow. When the conductor x is under the middle of the N-pole, as shown, the e.m.f. induced will be away from the reader and will be a maximum, since it is in the maximum field (we here assume the field flux to be maximum under the center of the pole). Since the load is pure resistance, the current I will be in phase with E, or is a maximum, at the same instant, x. Now, if the current is in phase with E, it will also be away from the reader, and will produce a circular armature flux as shown. It is noticed that the circular armature flux opposes the field flux on the left of the poles and strengthens the field flux on the right of the poles.

In the illustration the field flux is four lines uniformly distributed over the surface of the poles; with armature reactions the resultant flux emanating from the N-pole is still four lines, but they all emanate from the right side of x; the two armature lines destroy the two field lines on the left of x. In other words, the effect of armature reactions, pure resistance load, is to distort the field toward the right (into the trailing pole tip or in the direction of motion of the armature relative to the field, but not to weaken the field (ϕ). The numerical illustration given is misleading in that in an actual generator the field flux is many times stronger than the armature flux (reactions) even at full load.

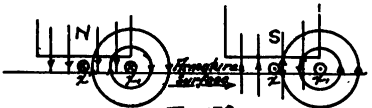


Fig 138

(*b*) *Resistance and Inductance Load.*—In Fig. 138, consider a two-pole generator. Again the voltage is a maximum when the armature conductor is in the position x (directly under the pole center), but since the load consists of R and L, I lags some distance behind E, or the current is not a maximum until the conductor on the armature reaches the point $x_{,}$. The current is again away from the reader, and therefore produces circular armature flux, as shown by the circular lines. It is clearly seen from the figure that in this case the armature flux (reactions) weakens the field. At no load (armature reactions = o) the flux emanating from the N-pole is four lines. With load as shown in Fig. 138, the useful flux emanating from the N-pole is only two lines.

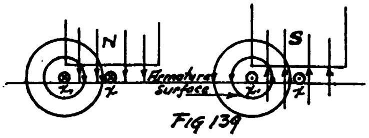


Fig 139

(*c*) *Resistance and Capacity Load.*—In Fig. 139 the voltage is again a maximum at x and away from the reader. Since the load consists of R and C, the current leads the voltage or is a maximum and away from the reader at $x_{,}$. The armature reactions are as shown, and it is clearly seen that they strengthen the field.

73. Summary.—(*a*) Armature reactions distort but do not weaken the field of a generator with pure resistance load (neglecting armature inductance).

(*b*) Armature reactions weaken the field of a generator, R and L load (lagging currents demagnetize the field).

(*c*) Armature reactions strengthen the field of a generator, R and C load (leading currents magnetize the field).

74. Clock Diagram for A.C. Generators under any Kind of Load.—In the following cases the letters used have the meanings given below:—

I = current flowing through armature and external circuit.
E = terminal voltage.
E_r = real induced e.m.f.
E_2 = virtual induced e.m.f.
ϕ_2 = flux required to produce E_2.
ϕ_r = armature reaction flux.
ϕ = flux produced by the field excitation.
R_a = resistance of armature.
L_a = inductance of the armature.

(*a*) *Pure Resistance Load.*—Consider the generator G connected to the pure resistance load R (Fig. 140). Since this is a series circuit, we will use I as the reference axis (Fig. 141). Since the load is pure resistance, the terminal voltage E is in phase with I; $R_a I$ is also in phase with I, and the real induced e.m.f. is the vector sum of E and $R_a I$.

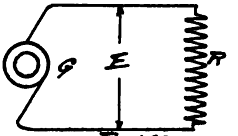


Fig 140

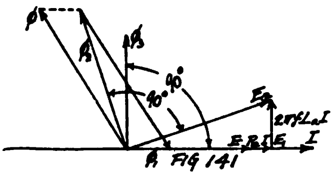


FIG 141

or $$E_r = E + R_a I \quad \text{(added vectorially)}$$

Since the armature has inductance $2\pi f L_a I$ is 90° ahead of I and the virtual induced e.m.f. (E_2) is ahead of I (Fig. 141) and is equal to the vector sum of E, $R_a I$, and $2\pi f L_a I$

or $$E_2 = E + R_a I + 2\pi f L_a I \quad \text{(added vectorially)}$$

this e.m.f. (E_2) has evidently been induced by the cutting of flux which is 90° ahead of E_2 or ϕ_2 (Fig. 141). ϕ_2, the total flux, is made up of two fluxes, one produced by armature reactions (amperes turn on the armature), ϕ, which is in phase with I, and the other ϕ produced by the field excitation and is the other component of ϕ_2.

(*b*) *Resistance and Inductance Load.*—Figs. 142 and 143 are exactly similar to Figs. 140 and 141, except that in this case the load is composed of R and L with a power factor of .5; that is, E leads I by 60° (Fig. 143).

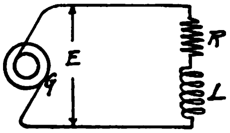


FIG 142

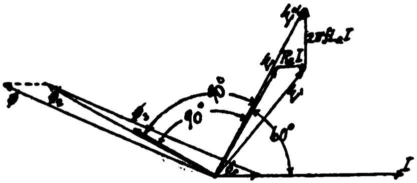


FIG 143

(*c*) *Resistance and Capacity Load.*—Figs. 144 and 145 are exactly similar to Figs. 140 and 141, except that in this case the load is composed of R and C, with a power factor −.5; that is, E lags behind I by 60° (Fig. 145).

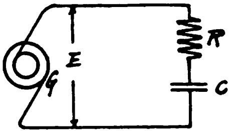


FIG 144

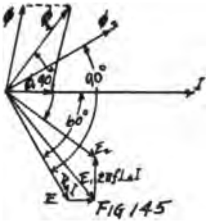


FIG 145

(See Steinmetz—"Elements of Electrical Engineering," "Self Induction," pp. 133–135.)

75. Conclusion.—A careful study of these three figures will show that with the same terminal voltage E, a much higher field excitation ϕ is needed with lagging, and a much lower field excitation is needed with leading current, than with non-inductive load.

Inversely with a constant field excitation ϕ the terminal voltage E of a generator drops off with non-inductive load, drops off much more with inductive load, and drops off much less, or may even rise, with capacity load.

In Figs. 141, 143, and 145 ϕ represents the relative position of the field pole to the armature.

On open circuit the field excitation required to produce a terminal voltage E would be ϕ_3, and is equal to $\frac{E}{E_2} \times \phi_2$ and is 90° ahead of E (see Figs. 141, 143, and 145), and this excitation is seen to be less than the field excitation ϕ with non-inductive load (Fig. 141).

76. Armature Self-induction.—The effect of self-induction is similar in every way to the effect of armature reactions.

The e.m.f. consumed in armature self-induction is 90° ahead of the current I and equals $2\,\pi f L_a I$. (See Fig. 141.)

The e.m.f. consumed by the armature reactions $\phi_{,}$ (Fig. 141) is 90° ahead of $\phi_{,,}$ and therefore also 90° ahead of I (since $\phi_{,}$ and I are always in phase with each other).

In general, both effects, self-induction and armature reactions, are grouped together under the term synchronous reactance (see art. 80).

77. The External Characteristics of a Generator.—The external characteristic of any generator is a curve plotted between the current in the external circuit (I) as abscissas and the terminal voltage E as ordinates.

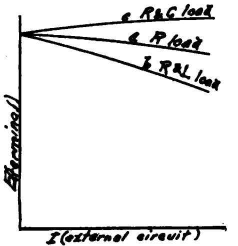


Fig 146

Fig. 146 represents the three external characteristic curves of a generator with the three loads (a) R (b) R and L (c) R and C and explained in art. 75.

78. Regulation.—The regulation of any D.C. or A.C. generator is the ratio of the maximum difference in terminal voltage from no load to the rated full load, to the rated full load voltage, or expressed as an equation:—

$$\text{Regulation in } \% = \frac{\text{no load voltage} - \text{full load voltage}}{\text{full load voltage}} \times 100$$

Example:—A 220-volt generator gives 242 volts at no load and 220 volts at full rated load:—

$$\%\ \text{Regulation} = \frac{242 - 220}{220} \times 100 = 10\%$$

Zero % regulation is always desired, but can never be attained.

79. Methods of Field Excitation.—Since all A.C. generators have permanent N- and S-poles, their fields must be excited by direct current. Since they can only supply alternating current, they cannot be self-exciting(as can a D.C. shunt generator), but must be excited from a separate D.C. machine, called the *exciter*.

From the discussion of characteristic curves it is learned that the terminal voltage tends to decrease as the load comes on, especially when the load is lamps (pure resistance), curve (a), Fig. 146, and also when the load is motors (R and L), curve (b), Fig. 146. These are loads generally met with in practice, and in each case it is very desirable to keep the terminal voltage constant, which means that E (induced) ($=2.22\ \phi fC$) must increase with increase of load. For this reason all methods of field excitation are arranged to increase the field strength as the load comes on.

There are three methods most generally used:—

1. *By Using a Shunt or Compound Wound Exciter.*—The connections in Fig. 147 represent diagrammatically the connections for armature and field of a single-phase four-pole A.C. generator. The field of the generator is excited by D.C. obtained from a small shunt generator (E) called the exciter (shown in the small rectangle).

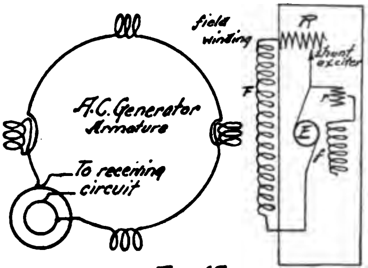


Fig 147

As load is put on the A.C. generator, in order to keep the terminal voltage constant, it is necessary to increase the field excitation. This can be done in two ways: first, by cutting out R, which is connected in series with the field of the A.C. generator; or, second, by cutting out some of the field rheostat (r) of the exciter. Neither of these methods is automatic. Compound wound exciters are also used. Exciters generally deliver 125 or 250 volts; this gives a convenient method of lighting the station.

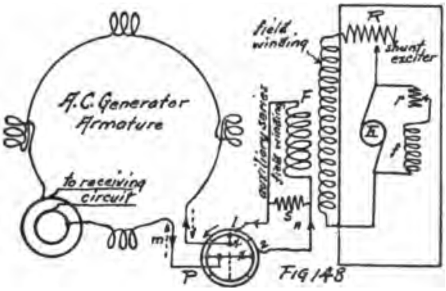


Fig 148

2. *By Using a Rectifier and Passing All the Main Current Through a Series Field Winding.*—This method (Fig. 148), has the

same excitation as in Fig. 147, but in addition has an auxiliary field winding on each pole entirely similar to the series field on a D.C. compound wound generator. This series field is connected in series with the main line, and is connected so that its excitation adds to the excitation produced by the main field, F, which is supplied by the exciter E. Now, as the load increases this auxiliary field, being in series with the line, also increases, and thus the total excitation increases and the terminal voltage can be made to remain nearly constant.

If A.C. were passed through this auxiliary field, it would add and then oppose the magnetization produced by F, and its net effect would be zero. For this reason the main current of the A.C. generator before passing through this auxiliary field must be rectified into direct current. This is accomplished by means of a device P (Fig. 148) called a rectifier or rectifying commutator. The rectifier consists of a commutator having p segments, where p is the number of field poles on the A.C. generator (4 in Fig. 148). These segments are insulated from each other and alternate segments are electrically connected together as shown. Two brushes rest on these segments, spaced a distance apart equal to the distance from center to center of adjacent commutator segments (or an odd multiple thereof), and these brushes deliver the rectified D.C. to the auxiliary field windings.

At a given instant suppose the current is flowing past (m), as shown by the full line; by carefully following through the rectifier it flows *up* through the auxiliary winding. One half-cycle later the current will be flowing past (m) as shown by the dotted arrows; but now the brush, which was before on the segment No. 1, is now on segment No. 2 (the segments have revolved one-fourth of a revolution), and therefore the current still flows *up* in the auxiliary field winding. The negative lobes of the sine waves have been reversed, provided the brushes pass from one segment to the next at the instant the current wave passes through zero. This position can only be determined by shifting the brushes back and forth, and when the sparking is a minimum, we know that this condition occurs. This method is used by the General Electric Company in their single-phase stationary field generators.

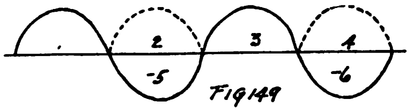


FIG 149

Fig. 149 shows the D.C. current wave produced; each negative lobe 5 and 6 of the A.C. current wave has been rectified so that we get the D.C. wave of the shape shown by the four lobes, 1, 2, 3, and 4, above the x-axis.

The compounding produced by this auxiliary field winding may be varied by varying the value of the shunt s (Fig. 148). The normal voltage when starting is obtained by regulating the field rheostat r or the rheostat R.

3. *By Using a Rectifier and Passing Some of the Main Current Through a Series Field Winding.*—Fig. 150 is entirely similar to Fig. 148, except that a series transformer is placed in series with the main line and only a part of the main current passes through the series field. This, of course, will reduce sparking at the rectifier brushes, which is always bad, and will prevent the high-voltage current of the armature circuit from coming in contact with the field windings, thus rendering their insulation less difficult and reducing the liability of shocks to the dynamo tender.

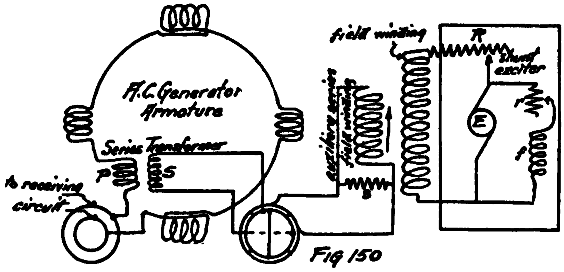


Fig 150

The main current passes through the primary P (see Fig. 150) of the transformer, and if the secondary S has ten times as many turns as the primary, it will carry one-tenth as much current as the primary (see chapter X). This arrangement has been employed by the Westinghouse Co., on their stationary field generators. The transformer is placed on one of the spokes of the armature spider.

The resistance R (Figs. 147, 148, and 150) is not used when each machine has its own exciter, because, carrying all the field current, it has too much I^2R loss. It is used only when the field is excited from some constant potential D.C. source. Because of the difficulty of commutating large currents, rectifiers are not used on very large generators.

The methods of Figs. 148 and 150 are fairly automatic for loads of variable resistance (incandescent lamps), but are not automatic for loads of variable resistance as well as variable inductance (motors), and for this purpose many other methods have been devised.

Best known of other methods is the Terrill regulator, which works on the principle of rapidly opening and closing a shunt circuit connected across the field rheostat of the exciter (for description see Franklin and Estey, second edition, "Alternating Currents," page 343).

Reference:—Sheldon, Mason, and Hausmann: "Alternating Currents," pp. 127–146.

80. Synchronous Reactance.—Armature self-induction, armature reactions, and armature resistance are all grouped together, and are called *synchronous impedance.* Armature reactance (self-induction, $2\pi fL_a$) and armature reactions are grouped together and called synchronous reactance.

Since reactance ($2\pi fL_a$) varies with the frequency f, the normal reactance will be obtained only when the machine is run at normal or name-plate frequency. The reactance thus obtained is called synchronous reactance.

Fig. 151 represents a diagram of connections for obtaining a synchronous reactance curve. The armature is short-circuited through an ammeter, A, and the generator is run at normal or synchronous speed. The field strength is varied by adjusting the field rheostat (F.R., Fig. 151) on the exciter until full load current flows through the armature, as indicated by the ammeter A. Then the switch, s, is opened, and, without changing the field ex-

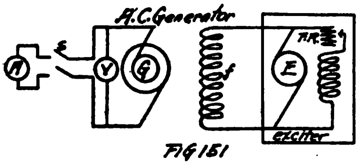


Fig 151

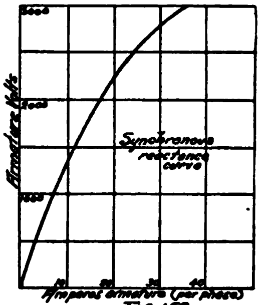


Fig 152

citation, the voltmeter, V, is read. When switch (s) is closed, clearly voltmeter will read zero, because it is short-circuited by the ammeter. While (s) is closed, there is a voltage generated in the armature because it has impedance and full load current is flowing through it. The value of this generated voltage is recorded by the voltmeter, V, when the switch is open, for the conductors on the armature are still cutting the same flux (the field excitation is unchanged) at the same rate (no change of speed) and the impedance drop is zero, since I is zero.

By varying the field excitation a series of ammeter and voltmeter readings may be obtained and a curve plotted between the voltmeter readings as ordinates and the ammeter readings as abscissas. The curve so obtained is a synchronous reactance curve. It is strictly a synchronous impedance curve, but the resistance is so small compared with the reactance that it is called the synchronous reactance curve.

The synchronous reactance is obtained by dividing the voltmeter readings by the corresponding ammeter readings.

Fig. 152 represents a synchronous reactance curve for a two-phase four-wire generator. For a diagram of connections of one phase for making this test, see Fig. 151.

81. Saturation.—A curve plotted between terminal volts and field current when the generator is run on open circuit is called a saturation curve.

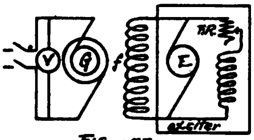


Fig 153

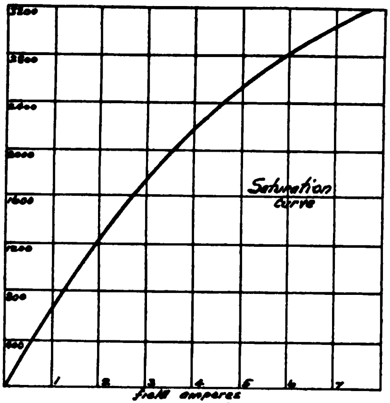


Fig 154

Fig. 153 gives a diagram of connections for obtaining the saturation curve (Fig. 154) of a single-phase generator. Successively increasing values on this curve may be obtained by cutting out the field rheostat on the exciter (F.R., Fig. 153). At low flux densities in the iron the reluctance of the air gaps is so much greater than the reluctance of the iron that the curve is nearly a straight line (reluctance of air is constant), but as the iron becomes more saturated its reluctance increases and becomes appreciable and the curve bends over.

82. Electromotive Force Method of Calculating the Regulation of a Generator.—This method may best be explained by working out an example.

It is required to determine the regulation of a 65 K.W. two-phase 2400-volt generator at full load current; resistance between terminals (one phase) is 5 ohms at 25° C.

(*a*) *With Unity Power Factor at Full Load Current.*—

$$I_{(\text{per phase})} = \frac{65000}{2 \times 2400} = 13.5 \text{ amperes}$$

In all regulation calculations it is customary to make them at 75° C.

Therefore since
$$R_{75} = R_{25}\,(1 + \alpha\, t)$$
$$R_{75} = 5\,[1 + (.004 \times 50)] = 6 \text{ ohms}$$

At unity power factor the terminal voltage E (2400) is in phase with the current, see Fig. 155. The $R_a I_a$ is also in phase with I, and $R_a I_a = 6 \times 13.5 = 81.0$ volts. From the synchronous reactance curve Fig. 152 we find $X_a I_a$ (Fig. 155) is 1550 volts.

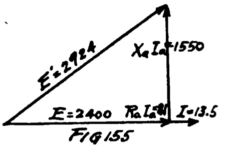


FIG 155

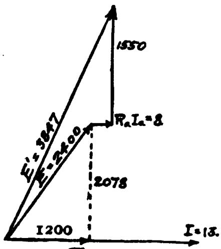


FIG 156

Therefore the total induced voltage or the no load terminal voltage is:—

$$E' = \sqrt{(2481)^2 + (1550)^2} = 2924 \text{ volts}$$

and the full load regulation in % at unity power factor is:—

$$\frac{2924 - 2400}{2400} = 21.8\%$$

(*b*) *With a Power Factor of .5 at Full Load Current.*—In this case the terminal voltage, E, is ahead of I by the angle whose cos is .5 (60°), Fig. 156. Therefore the two components of E are:—

$$2400 \cos 60° = 1200 \text{ volts}$$
$$2400 \sin 60° = 2078 \text{ volts}$$

as before $R_aI_a = 81$ volts, in phase with I
as before $X_aI_a = 1550$ volts, 90° ahead of I

$$\therefore \quad E' = \sqrt{(1281)^2 + (2078+1550)^2} = 3847 \text{ volts}$$

and % regulation at .5 power factor

is $$\frac{3847-2400}{2400} = 60.3\%$$

83. Magneto-motive-force Method of Calculating the Regulation of a Generator.—The exciting ampere turns on the field may be thought of as consisting of two parts. One part produces the flux (ϕ) required to generate the e.m.f. in phase with I (2481 in Fig. 155). The other part produces the flux (ϕ') required to generate the e.m.f. 90° ahead of I (1550 in Fig. 155). Since these two fluxes, ϕ and ϕ', are 90° apart, so will these two sets of ampere turns be 90° apart.

Example:—It is required to work the above problem (see Art. 82) by the magneto-motive-force (m.m.f.) method.

(*a*) *With Unity Power Factor at Full Load Current.*—From the saturation curve (Fig. 154) the field current required to produce 2481 volts (Fig. 155) = 4.7 amperes.

The field current required to produce 1550 volts (Fig. 155) = 2.55 amperes, but these two currents are 90° apart.

$$\text{Therefore the total field current } I = \sqrt{(4.7)^2 + (2.55)^2} = 5.35 \text{ amperes}$$

From the saturation curve (Fig. 154), the no load voltage corresponding to a field excitation of 5.35 is 2620 volts.

$$\text{Therefore the \% regulation} = \frac{2620-2400}{2400} = 9.16\%$$

(*b*) *With a Power Factor of 5. at Full Load Current.*—From the saturation curve (Fig. 154) the field current required to produce 1281 volts (Fig. 156) = 2 amperes; the field current required to produce 3628 volts (2078+1550, Fig. 156) = 16 amperes.

$$\text{Therefore the total field current } I = \sqrt{(2)^2 + (16)^2} = 16.12 \text{ amperes}$$

From the saturation curve (Fig. 154), the no load voltage corresponding to 16.12 amperes is 3640 volts.

$$\text{Therefore the \% regulation} = \frac{3640-2400}{2400} = 51.7\%$$

The actual regulation obtained by running the machine with full load and unity power factor was 14.6%.

This example shows that the e.m.f. method gives regulation generally too poor (21.8%), and that the m.m.f. method gives a regulation generally too good (9.16%), so that a fairly good result is obtained by taking a mean of the two results (15.5%).

In calculating regulation it is well to reduce everything to terms of phase values, as shown by the two following examples:—

Example 1:—A 2000-volt 2000 K.W. three-phase Y-connected generator (Fig. 157).

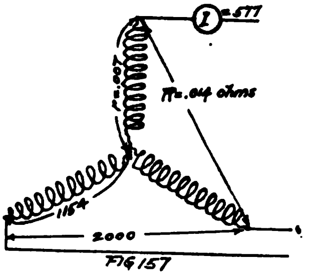


FIG 157

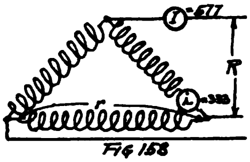


FIG 158

$$I_{(\text{line})} = \frac{KW}{\sqrt{3}\,E} = \frac{2{,}000{,}000}{\sqrt{3}\times 2000} = 577 \text{ amperes}$$

$$i_{(\text{per phase})} = I = 577 \text{ amperes}$$

$$E_{(\text{line})} = 2000 \text{ volts}$$

$$e_{(\text{per phase})} = \frac{E}{\sqrt{3}} = \frac{2000}{\sqrt{3}} = 1154 \text{ volts}$$

$$R_{(\text{between terminals})} = .014 \text{ ohms}$$

$$r_{(\text{per phase})} = \frac{R}{2} = \frac{.014}{2} = .007 \text{ ohms}$$

Example 2:—A 2000-volt 2000 K.W. three-phase Δ-connected generator (Fig. 158).

$$I_{(\text{line})} = \frac{KW}{\sqrt{3}\,E} = \frac{2{,}000{,}000}{\sqrt{3}\times 2000} = 577 \text{ amperes}$$

$$i_{(\text{per phase})} = \frac{E}{\sqrt{3}} = \frac{577}{\sqrt{3}} \quad 333 \text{ amperes}$$

$$E_{(\text{line})} = 2000 \text{ volts}$$

$$e_{(\text{per phase})} = E = 2000 \text{ volts}$$

$$R_{(\text{between terminals})} = \frac{1}{\frac{1}{r}+\frac{1}{2r}} = \frac{2}{3}r = .014 \text{ ohms}$$

$$r_{(\text{per phase})} = \frac{3}{2}R = \frac{3}{2}\times .014 = .021 \text{ ohms}$$

84. Efficiency.—From the report of the Committee on Standardization of the American Institute of Electrical Engineers, we quote the following: "The efficiency of an apparatus is the ratio of its net power output to its gross power input."

In determining efficiency, unity power factor should be used except in machines which naturally have a phase difference, as induction motors.

When a machine has auxiliary apparatus, such as an exciter, the power lost in such apparatus should not be charged to the machine efficiency, but to the plant efficiency.

The efficiency may be obtained by measuring all the losses and adding them to the output to obtain the input, or they may be subtracted from the input to obtain the output. In all electrical apparatus it is easier to measure the electrical than the mechanical power; therefore in generators it is easier to obtain the efficiency by using the formula:—

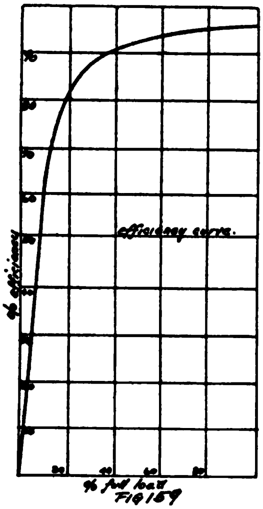


FIG 159

$$\text{efficiency} = \frac{\text{output}}{\text{output} + \text{losses}}$$

and in motors the formula:—

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}$$

Output in the case of the generator and input in the case of the motor are both easily measurable by using a wattmeter.

85. Generator Losses.—(*a*) Bearing friction and windage.

(*b*) Hysteresis and eddy current losses in all metallic parts.

(*c*) I^2R losses in armature.

(*d*) Load losses, that is, eddy currents and hysteresis due to the current flowing in the armature under load, equal roughly to about one-third of losses under (b).

(*e*) Collector ring and brush contact resistance (negligible).

(*f*) Field excitation—in separately excited machines (generators and synchronous motors) the I^2R of the field coils only—in self-exciting machines (rotary converters) the loss in the field rheostat should be included.

PROBLEMS.

130. Make a development similar to Fig. 132 for:—
    (*a*) 4-pole, 12 slots per pole, single-phase generator.
    (*b*) 4-pole, 12 slots per pole, two-phase generator.
    (*c*) 4-pole, 12 slots per pole, three-phase generator.

131. Make a development similar to Fig. 132 for:—
    (*a*) 6-pole, 3 slots per pole, single-phase generator.
    (*b*) 6-pole, 3 slots per pole, two-phase generator.
    (*c*) 6-pole, 3 slots per pole, three-phase generator.

132. On a single-phase generator there are 40 poles, each 5 inches by 9 inches. The flux density $B = 50,000$ lines per square inch. With a speed of 180 r.p.m. what will be the e.m.f. generated if there are 4480 conductors on the armature? *Ans.*:—13,426.6 volts.

133. In problem 132 what must be the speed in order that the voltage at the terminals may be 5500 volts? *Ans.*:—73.7 r.p.m.

134. A two-phase generator has the following data: 40 poles, each 6 inches by 8 inches; $B = 60,000$ lines per square inch; frequency 60 cycles; conductors in each phase 3360. Find:—
    (*a*) Voltage per phase.
    (*b*) Voltage across each pair of wires if connected three-wire.
    (*c*) Speed.
    Ans.:—(*a*) 12,889.5; (*b*) 18,175; (*c*) 180 r.p.m.

135. In problems 132 and 134 find the number of conductors in each slot if there are six slots per pole? *Ans.*:—28 in both cases.

136. Allowing a total flux per pole of 10^6 lines and assuming a speed of 100 r.p.m. and 32 conductors in each slot, find the e.m.f. generated in each phase of a 30 pole, 6 slots per pole: (*a*) single-phase, (*b*) two-phase, (*c*) three-phase generator. *Ans.*:—(*a*) 2131.2; (*b*) 1598.4; (*c*) 1065.6.

137. In problem 136 (*c*) find the line voltage when the armature is:—
    (*a*) connected Y.
    (*b*) connected △.

138. In the formula $E_{(r.m.s.)} = \frac{2.22\ \phi f C}{10^8}$ make the necessary correction to allow for all the conductors of a phase being in different slots in the case of:—
    (*a*) 6 slots per pole two-phase winding.
    (*b*) 6 slots per pole one-phase winding.
    (*c*) 3 slots per pole one-phase winding.
    (*d*) 3 slots per pole three-phase winding.

139. The distance from center to center of adjacent poles of a generator is 10½ inches. How far apart must two armature conductors be placed so that there may be a phase difference of 50° in the e.m.f.'s induced in the respective conductors? *Ans.*:—2.917 inches.

140. Calculate the regulation of a 65 K.W. two-phase 2400-volt generator at three-fourths full load current when the power factor of the receiving circuit is .8 (Figs. 152 and 154, as this is the same generator).
    (*a*) By the e.m.f. method.
    (*b*) By the m.m.f. method.
    Ans.:—(*a*) 38.3%; (*b*) 18.8%.

141. If the load in the preceding problem were replaced by a capacity load of the same power factor, what would be the regulation by both methods? *Ans.*:—(*a*) 17.1% (no load e.m.f. = 1990 volts); (*b*) 17.1%.
    (*Note:*—Answers to problems 140 and 141 are not accurate because of the difficulty of interpolating accurately on the curves Figs. 152 and 154.)

142. By means of a diagram similar to Fig. 137 determine the effect of armature reactions on the field of a generator if the load were purely inductive. How will the external characteristic compare with those in Fig. 146?

143. Do problem 142 under the assumption that the load is purely capacity.

CHAPTER IX.

SYNCHRONOUS MOTORS.

Fig. 160 is a diagram of connections for the operation of two A.C. generators, A and B, in parallel, so that the power delivered to the receiving circuit is the sum of the power delivered by each machine.

Fig. 161 shows a diagram of connections for operating one machine, A, as a generator and the other machine, B, as a motor. Since the generator A and the motor B are generally some distance apart from each other, each machine has to have its own exciter or other source of field excitation.

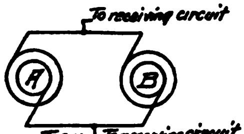


Fig 160

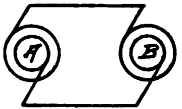


Fig 161

Fig. 160 forms a closed circuit, exactly as does Fig. 161, and the interaction of two generators in parallel is in so many respects so similar to the operation of a generator driving a synchronous motor that the two discussions will be given in this chapter.

86. Clock Diagram for a D.C. Motor.—Let OE (Fig. 162) represent the impressed e.m.f. on a shunt motor. The b.e.m.f. (oe) is, of course, opposite to OE, or 180° from OE, and therefore the resultant e.m.f. OR = OE + oe (vectorially).

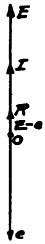


Fig 162

Since it is a D.C. system, the current (I) will be in phase with this resultant e.m.f. (OR) (pure resistance) and $I=\frac{E-e}{R}$ (numerically), where R is the resistance of the armature of the motor.

The electrical input is OE×I; neglecting hysteresis, eddy currents, windage, and friction, the mechanical output is OE×I; neglecting I^2R losses, OE×I = oe×I.

If the load be suddenly increased, the motor slows up; therefore oe decreases (since oe αNcn); and since $I=\frac{E-e}{R}$, I increases, and input, which equals OE×I, also increases.

If the load be suddenly decreased, the motor speeds up, oe increases, I decreases, and input, which equals OE×I, decreases. Thus we see that a D.C. motor will operate under varying loads, and adopts itself to these loads by a change of speed.

The A.C. current in any armature conductor on the motor is in one direction when passing under N-poles and in the opposite direction when passing under S-poles; otherwise the torque would not be in the same direction in both cases; and if because of an increase in load the motor slows down, this condition still holds because the brushes are not sliding over the commutator bars so rapidly and the frequency in the armature has been reduced in exact proportion to the reduction of speed. Because of this relation of armature and field, all D.C. motors are synchronous motors and will operate only when running in synchronism. D.C. motors have a frequency in their armatures which varies in proportion to their speed and produced automatically by the commutator.

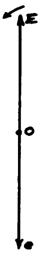


Fig 163

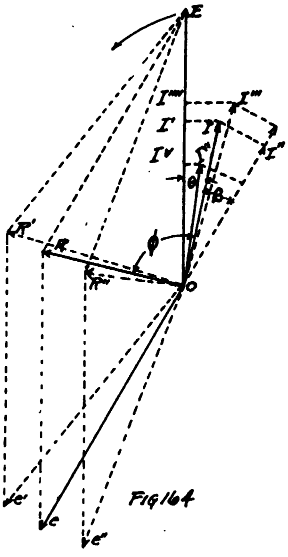


Fig 164

The main difference between an A.C. generator and a D.C. generator is that the A.C. generator has no commutator. Any D.C. generator can be operated as a D.C. motor; and, for exactly the same reason, any A.C. generator will run as a synchronous motor.

87. Clock Diagram for an A.C. Motor.—Since all synchronous motors run on constant frequency circuits and have no commutator, in order for them to stay in synchronism they can have but one speed (n). The b.e.m.f. = oe (Fig. 163) = $2.22\phi fC$ (see equation 60).

In this formula f and C are constant, and therefore by adjusting the excitation (field rheostat on the exciter) oe may be made to have any value.

Let us suppose that the field strength (ϕ) of the synchronous motor has been so adjusted that in Fig. 163 oe = OE, the impressed e.m.f. Under these conditions the resultant e.m.f. OR is zero, and therefore I is zero and the input $OE \times I = \text{zero}$.

Since the machine must run in synchronism, some energy is required to run it, and the input being zero the machine temporarily slows up and oe gains on OE, so they have the relative positions shown in Fig. 164. The motor is still running in synchronism, but the phase angle between OE and oe has been reduced from 180° to 165°.

The resultant voltage OR is now the vector sum of oe and OE.

The current $OI = \frac{OR}{\sqrt{R^2+X^2}}$ and lags behind OR by the angle ϕ, where $\tan \phi = \frac{X}{R}$

X is the reactance of the armature of the motor and
R is the resistance of the armature of the motor
The electrical input $= OE \times I \times \cos \theta = OE \times I'$
Neglecting hysteresis, eddy currents, and friction, the mechanical output is

$$oe \times I \times \cos \beta = oe \times I''$$

Neglecting I^2R losses, $OE \times I' = oe \times I''$

If the load be suddenly increased, the motor will temporarily slow up, oe will take the position oe′ (Fig. 164), the resultant will increase to the value of OR′ and the current $OI = \frac{OR'}{\sqrt{R^2+X^2}}$ will increase to OI‴, and therefore the input $OE \times I''''$ will increase just sufficiently to take care of the added load.

If the load be suddenly decreased, the motor will temporarily speed up, oe will take the position of oe″, OR will be reduced to OR″, and the current $OI = \frac{OR''}{\sqrt{R^2+X^2}}$ will decrease to $OI^{v\prime}$ and therefore the input $= OE \times I^{v}$ will decrease just sufficiently to balance the reduced load.

From this discussion we can see that a synchronous motor will operate satisfactorily and adapt itself to varying loads by a slight phase swinging of oe with respect to OE.

If the load is increased too much, the motor will fall out of synchronism and stop, and the heavy current produced will throw out the circuit breakers. (For a discussion of stability of synchronous motors, see Sheldon, Mason, and Hausmann, "Alternating Current Machines," eighth edition, 1909, pp. 249–256; also Franklin and Esty, "Alternating Currents," second edition, pp. 141–150.)

88. Hunting.—Referring to Fig. 164, it will be noticed that when the load is increased the b.e.m.f. of the motor oe falls back to the position oe′. As oe falls back toward oe′, the input increases, but does not take care of the increased output until it reaches oe′; but when passing oe′, the motor is slowing up; at this point, oe′, the input is only able to take care of the output; therefore the motor continues to slow up, but now the input has increased more than enough to take care of the output, so that the motor speeds up. In other words, when the b.e.m.f. vector oe has to take up a new position with respect to the impressed e.m.f. OE, it does not immediately change from the old position (oe) to the new position oe′, but oscillates back and forth, and finally settles in the new position oe′ (Fig. 164). This oscillation of oe′ is known as hunting.

89. Causes of Hunting.—1. Variation of load on the motor.

2. Variation of impressed frequency due to engine governor.

3. Variation of impressed e.m.f. in value due to the sudden application of other motor loads on the same generator.

4. Inertia of moving element of the motor (armature in revolving armature type).

5. Electrical inertia—high inductance of armature of motor.

90. Methods of Preventing Hunting.—1. Close regulation of the speed of the prime mover (especially steam-engines) for the generator; this is the most important method.

2. A fly-wheel loosely attached to the motor shaft so as to slip back and forth as the motor's speed increases and decreases.

3. The use of copper bars (Fig. 165) set into the pole faces of the field frame parallel to the shaft, the ends of these bars being short-circuited by end rings of copper. (See General Electric bulletin No. 4847 on belt-driven alternators.)

This method operates as follows: Because of the hunting, the torque exerted by the field on the armature increases and decreases; this variation in torque is accompanied by a corresponding shifting of the flux back and forth across the pole face. This shifting flux induces heavy currents in the short-circuited copper bands, and these currents tend to prevent the shifting of the flux, thus damping out the oscillations. (The same principle that is used in having a copper frame for the coils of D.C. ammeters and voltmeters to make them "dead beat.")

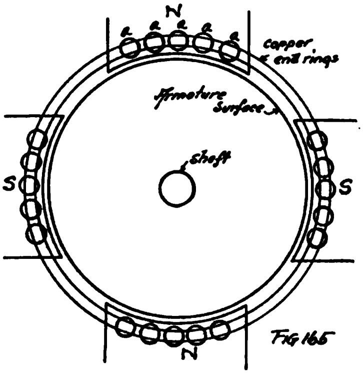


Fig 165

91. The Action of an Overexcited Synchronous Motor.—Fig. 166 is an exact duplication of Fig. 164. Suppose we increase the excitation of the motor so that the b.e.m.f. is increased from oe to oe′ (Fig. 166); then the resultant e.m.f. will clearly be OR′ and the current will be OI′, and, as before, will lag behind OR′ by the angle ϕ (tan $\phi = \frac{X}{R}$).

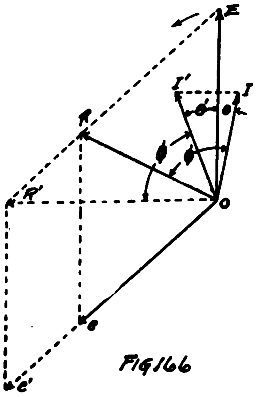


Fig 166

With normal excitation (oe) the power factor of the circuit is cos θ, which is positive, and OI lags behind OE; with the increased excitation (oe′) it is noticed that the power factor cos θ' is negative and OI′ leads OE. If oe is greater than OE, the motor is said to be overexcited; therefore if a synchronous motor is sufficiently overexcited, it will produce a leading current. Synchronous motors are sometimes overexcited and placed at the end of the line in parallel with other motors having a poor power factor; they thus improve the power factor of the transmission line and thus the regulation of the generating station. When so used, they are generally run without load and are called "*synchronous compensators.*"

92. The Parallel Operation of A.C. Generators.—In Fig. 164

suppose the synchronous motor should be belted to a prime mover and its speed temporarily increased; under these conditions the machine is no longer a synchronous motor, but a generator.

In Fig. 167 OE represents the bus bar e.m.f. and oe the e.m.f. of the generator. The resultant e.m.f. is OR and the current $I = \frac{OR}{\sqrt{R^2+X^2}}$ and lags behind OR by the angle ϕ ($\tan \phi = \frac{X}{R}$).

The total electrical output of the generator is $oe \times I''$.

The useful power transmitted to the bus bars is $OE \times I'$.

It is interesting to compare Figs. 164 and 167. In Fig. 164 in the expression $oe \times I''$, I'' is obtained by dropping a perpendicular from I on oe produced backward, which means that machine oe is running as a motor, while in Fig. 167, in the expression $oe \times I''$, I'' is obtained by dropping a perpendicular from I on oe, which means that machine oe is running as a generator.

When the perpendicular from the end of I falls on point o, then the expression $oe \times I''$ becomes zero, and the machine is neither a motor nor a generator; it is running empty, because:—

$$\text{Power} = oe \times OI \times \cos\theta \text{ (Fig. 167); and since } \theta \text{ is } 90^\circ,$$
$$\text{Power} = oe \times OI \times O = O$$

The same trouble due to hunting exists with generators in parallel as exists in synchronous motor operation, and the same methods of prevention are in vogue. (For further discussion of generators in parallel see Steinmetz, "Elements of Electrical Engineering," 1902, pp. 154–161; also "A Practical Consideration of Parallel Operation of Alternating Current Generators and Other Synchronous Machinery," by B. P. Coulson, Jr., General Electric Review, Sept., 1911, p. 418.)

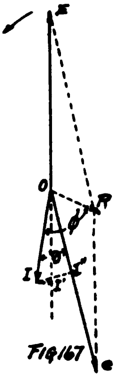


FIG 167

93. Armature Reactions in Synchronous Motors.—Since the current in a synchronous motor is practically 180° away from where it is when running as a generator, the armature reactions have exactly the opposite effect. Thus if Fig. 137 were correctly to represent the reactions in a motor, the flow of currents in the armature conductors should be exactly reversed from the way they are shown in the figure, which means that the field would be distorted into the leading pole tip, and what is the dense portion of the pole would be on the left instead of on the right as shown. A similar difference exists in Figs. 138 and 139.

Summary.—(*a*) Armature reactions distort but do not weaken the field of a synchronous motor with unity power factor (equivalent to pure resistance load on a generator).

(*b*) Armature reactions strengthen the field of a synchronous motor with lagging current (equivalent to resistance and inductance load on a generator).

(*c*) Armature reactions weaken the field of a synchronous motor with leading current (equivalent to resistance and capacity load on a generator).

94. **Methods of Starting.**—The methods of starting are similar to those given for rotary converters, and since the commonest method involves the induction motor principle, they will be given in Chapter XIII.

95. **Methods of Synchronizing.**—Consider two 110-volt machines, A and B, Fig. 168. We will suppose A is running as a sixty-cycle generator, and with its field fully excited it delivers 110 volts at the switch ss; before s can be closed and machine B operate satisfactorily as a synchronous motor two conditions must be fulfilled:—

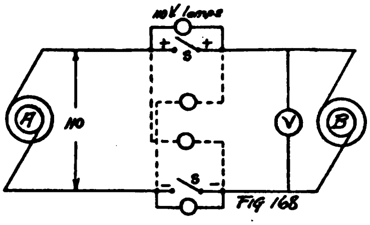


FIG 168

First: Machine B must have the same frequency and be exactly in synchronism with A.

Second: They must both have approximately the same e.m.f. A device which will indicate whether the first of these two conditions has been fulfilled before the switch ss is closed is called a synchronizing device or synchroscope or synchronizer.

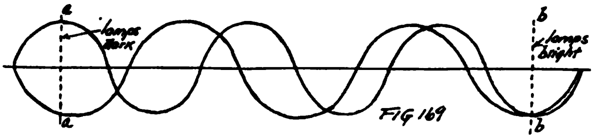


FIG 169

96. **The Use of Lamps for Synchronizing.**—Connect two 110-volt incandescent lamps across the switch ss as shown, Fig. 168; with switch ss open bring machine B up to speed by connecting it to some independent source of power (as a small induction motor)

and excite its field. Adjust its field strength until its voltage is about 110 volts, as read on a voltmeter, v, Fig. 168. Now, if machine B is running at fifty-eight cycles per second and A is making sixty, then the lamps will brighten and go out twice each second.

When the 2 e.m.f.'s are opposing each other (aa, Fig. 169), the resultant e.m.f. $= 110 - 110 = 0$, and the lamps are dark; when the e.m.f.'s add (bb, Fig. 169), the resultant e.m.f. $= 110 + 110 = 220$ (therefore two 110-volt lamps are needed, as the armatures A and B, Fig. 168, have negligible resistance) and the lamps are bright.

From Fig. 164 or 167 the e.m.f.'s should be about 180° apart, so the proper time for closing the switch ss is when the lamps are *dark.*

If the lamps are connected as shown by the dotted lines, Fig. 168, the proper time for closing the switch ss is when the lamps are *bright.* When lamps are used, it is generally customary to synchronize "lamps dark."

To avoid the use of twenty lamps in series on a 1100-volt generator, two small 10 to 1 step-down transformers are used, as shown in Fig. 170. To be sure that the transformers are connected correctly the first time, it is customary to raise the brushes on the motor B and close switch ss; then both transformers obtain power from the same source, A, and the polarity is as shown. If under these conditions the lamps show dark, with such connections, you will always synchronize "dark"; if the lamps show bright, reverse either primary or either secondary, and then the connections are right for synchronizing "dark."

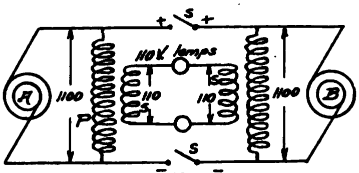


FIG 170

The advantage of synchroscopes or synchronizers is that they tell when the machines are in synchronism, and also which machine is running too fast; this last fact lamps will not disclose.

This discussion of synchronizing applies as well to generators in parallel, to rotary converters, and to motor generator sets.

For synchronizing polyphase machines see Westinghouse Bulletin, No. 7038, p. 44.

The discussions in this entire chapter have been devoted to single-phase generators and synchronous motors. The same discussions hold equally well when applied to any one phase of a polyphase generator or motor.

Single-phase motors will run in either direction, while polyphase motors with given connections to the supply mains will run in one direction only. (See art. 69.)

PROBLEMS.

144. In a clock diagram similar to Fig. 164 there are the following values: $OE = 2200$, $oe = 2100$, $\angle eoE = 180^\circ$, $X = 1$ ohm, and $R = 2$ ohms. Find:
    - (*a*) Power delivered to motor.
    - (*b*) Power delivered by motor.

    Ans.:—(*a*) 87,100 watts; (*b*) 83,200 watts.

145. A synchronous motor has an e.m.f. of 1100 volts and is driven from 1000-volt supply mains; the resistance of its armature is 1 ohm, its reactance .58 ohm.
    - (*a*) What is the value of the current?
    - (*b*) What is the component of this current which is 90° ahead of the supply e.m.f.?
    - (*c*) What capacity of condensers would take the same amount of leading current from the 1000-volt mains, at a frequency of sixty cycles per second (angle Eoe, Fig. 164, $= 167^\circ\ 44''$)?

    Ans.:—(*a*) 211.4 amperes; (*b*) 206.6 amperes; (*c*) 548 microfarads.

    (*Note:*—$OR = oe^2 + OE^2 + 2OE \times oe \cos \angle eoE$ in Fig. 164.)

CHAPTER X.

TRANSFORMERS.

97. General.—The A.C. transformer consists of one magnetic circuit linked with two electrical circuits. The electrical circuit which receives the energy (motor action) is called the primary (P); the electrical circuit which delivers the energy (generator action) is called the secondary (S) (see Fig. 51).

The conventional method of representing a transformer is shown in Fig. 171. The primary is connected to the generator (G) and the secondary is connected to the receiving circuit. (Fig. 171 represents two transformers.)

The purpose of a transformer is to change the e.m.f. of an A.C. circuit.

Example:—A generator generates energy at 10,000 volts. It is required to transmit this energy 50 miles at 100,000 volts. By transmitting at 100,000 volts instead of 10,000, the current is reduced ten times and the I^2R loss in the 50 miles of line is reduced one hundred times. A "step-up" transformer is installed in the generating station which receives current in its primary coil at 10,000 volts (Fig. 171) and delivers current from its secondary at 100,000 volts, and hence called a step-up transformer. At the receiving end of this line is installed a "step-down" transformer which receives current on its primary at 100,000 volts and delivers current from its secondary at 2200 volts, hence called a step-down transformer.

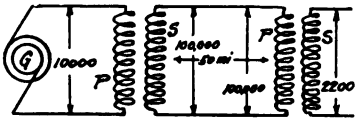


FIG 171

Step-up transformers have fewer turns in their primaries than on their secondaries, and are generally used at the generating end of a line. Step-down transformers have more turns on their primaries than on their secondaries, and are generally used at the receiving end of the line (sub-stations).

98. Construction.—There are two general types of transformers in general use: (1) the core type, (2) the shell type.

The Core Type.—Fig. 172 represents a core type transformer. There are two primary coils and two secondary coils, one of each placed on each leg. This arrangement reduces leakage and halves the dangerously high potential between two coils. There is but one magnetic path, which is through the iron. All transformer cores are laminated like armatures of generators, to cut down eddy current and hysteresis losses, also known as the iron losses.

The Shell Type.—Fig. 173 represents a shell type transformer. In this case there is shown one primary and two secondaries sandwiched between each other to reduce leakage. In this transformer there are magnetic paths, as shown by the arrows (Fig. 173).

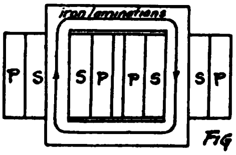


Fig 172

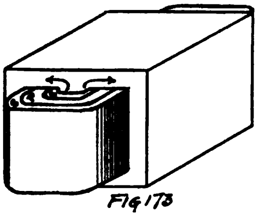


Fig 173

In Fig. 172 the electric circuit surrounds the magnetic circuit.

In Fig. 173 the magnetic circuit surrounds the electric circuit.

Makers of the shell type claim for it better regulation (see art. 78) and cooler running. The iron is cooler than in the core type. The ageing of the iron, which means an increase of the hysteretic constant (see art. 105), is believed to be aggravated by heat. However, the main object in keeping a transformer cool is not to save the iron, but to protect the insulation; and since the core type has less iron and therefore less iron loss, the advantages do not entirely favor either type.

In very high-tension transformers the primary and secondary may consist of eight or ten separate coils, sandwiched in between each other. This reduces the tension on any one high-tension coil to one-eighth or one-tenth of the line e.m.f. on the high-tension side.

Transformers are being manufactured for voltages considerably in excess of 100,000 volts. (For details of construction, methods of insulation, shape of laminations, etc., see Sheldon, Mason, and

Hausmann, "Alternating Currents," pp. 189–199; also General Electric and Westinghouse transformer bulletins. For General Electric type H transformer see General Electric bulletin No. 4802. For three-phase transformers see Westinghouse bulletin 1151. For high-tension transformers see Westinghouse bulletin 1079.)

99. Cooling of Transformers.—Transformers are generally cooled in any one of three ways:—

(1) Oil cooled: This is the method generally used for cooling small pole type transformers. The case is filled with oil to a level slightly above the coils. The oil also adds to the quality of the insulation and strengthens it against lightning and other static discharges.

(2) Air blast: The transformers are arranged in a row (transformer bank) over a pit. Air is driven into this pit and up through each transformer by a blower, generally driven by an induction motor.

(3) Oil and water: The oil is in contact with the coils as in method 1, but it is kept cool by being placed in contact with a large coil of pipes, through which cold water is constantly pumped.

100. Theory of a Perfect Transformer.—(*a*) *With pure resistance load.*—We will first consider the transformer to be unloaded or with the secondary on open circuit. Since the primary is connected to an A.C. generator, there will be an alternating flux (ϕ) produced in the iron, which is taken as the reference axis in the clock diagram (Fig. 174). This flux (ϕ), or magnetization of the iron, as it is called, is produced by a current (m) flowing in the primary which is in phase with ϕ and called the magnetizing current. Since ϕ is alternating in value, it will induce an e.m.f. in every coil that encircles

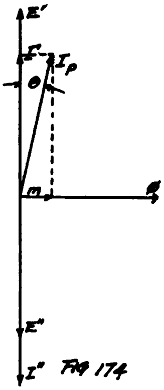


Fig 174

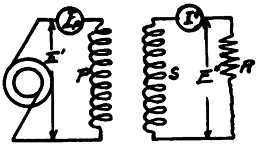


Fig 175

the iron; that is, it will induce an e.m.f. in both primary and secondary, which e.m.f. will be 90° behind ϕ. The e.m.f. so induced in the secondary coil is called E″, Fig. 174. In order that there may be equilibrium, there must be impressed on the primary from the generator an equal and opposite e.m.f. E′ (Fig. 174).

This completes the no-load discussion of the transformer (compare with Fig. 56). Now consider the transformer to have a load (pure resistance) as shown in Fig. 175. Then clearly a current I″ will flow in the secondary, and this current will be in phase with E″ (pure resistance load, see Fig. 174). But there are now I″ N″ (N″ = number of turns of wire on the secondary coil) ampere turns on the secondary which will produce flux through the iron; in order that there will be equilibrium there must be an equal and opposite number of ampere turns I′ N′ (N′ = number of turns of wire on the primary coil) on the primary, which means that I′ (Fig. 174) must be 180° from I″.

There are now two currents in the primary:—

(*a*) The magnetizing current, m (the only current at no load) and

(*b*) The load current, I′, so that the total current in the primary is the vector sum of I′ and m, or I_p. The power factor in the secondary is unity; the power factor in the primary is cos θ (Fig. 174), which is less than unity, due to the fact that m is 90° from E′. The no-load power factor on the primary is, of course, zero.

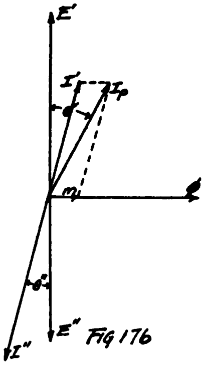


Fig 176

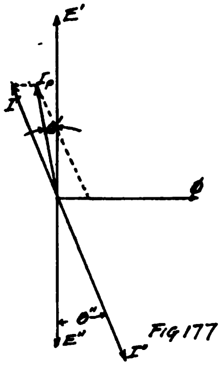


Fig 177

(*b*) *With resistance and inductance load.*—Fig. 176 is a clock diagram for a transformer with a R and L load. The no-load conditions are, of course, unchanged. But now I″ lags behind E″

by θ''° and I′ is again 180° from I″. In this case the angle θ' is larger than θ''; therefore as in case (*a*) the primary has the poorer power factor.

(*c*) *With resistance and capacity load.*—Fig. 177 is a clock diagram for a transformer with R and C load. In this case I″ leads E″ by θ''°.

101. Fundamental Equation of the Transformer.—From equation (15) we have $E_{(r.m.s.)} = 4.44\ \theta_{(max)}\ fn$, where E is the e.m.f. induced in any coil of wire by the changing flux through the coil when excited by alternating current. If we call N″ the number of turns on the secondary, equation (15) becomes:—

$$E'' = 4.44\ \phi_{(max)}\ fN'' \qquad (62)$$

and in the same way

$$E' = 4.44\ \phi_{(max)}\ fN' \qquad (63)$$

dividing (62) by (63) we get

$$\frac{E''}{E'} = \frac{N''}{N'} \qquad (64)$$

or the primary and secondary e.m.f.'s in a transformer are directly proportional to the ratio of the number of turns.

From the discussion given in the preceding article we see that:—

$$N''I'' = N'I'$$

or

$$\frac{I''}{I'} = \frac{N'}{N''} \qquad (65)$$

or the primary and secondary currents in a transformer are inversely proportional to the ratio of the number of turns.

102. The Effect of the Primary and Secondary Resistance.—(*a*) Pure resistance load. As in Fig. 174, E″ and E′ are the voltages at the terminals of secondary and primary if the resistance of the windings is neglected. But since the secondary has a resistance R″, there will be a drop I″ R″ proportional to I″ (R″ being constant) and 180° from I″ (see Fig. 178), so that the terminal voltage E_s is less than the induced voltage E″ (the terminal voltage of a D.C. generator is less than the induced voltage by the IR drop in the armature, and the secondary is similar to a generator).

The necessary voltage impressed on the primary would be E′ if the resistance of the primary coil could be neglected; but the e.m.f. from the generator has to overcome, not only E′, but a resistance drop in the primary I_pR', which is proportional to I_p and in phase with I_p. So the total impressed e.m.f. on the primary coil from the generator is E_p, which is larger than E′ (the terminal voltage of a D.C. motor is greater than the b.e.m.f. by the IR drop in the armature, and the primary is similar to a motor).

Fig. 179 shows the phase relations R and L load.

Fig. 180 shows the phase relations R and C load.

These two figures are entirely similar in construction to Fig. 178, and by carefully studying the values of θ' and θ'' in each case,

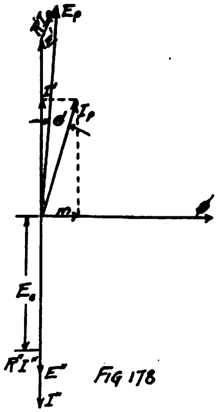


Fig 178

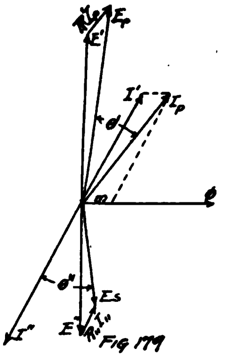


Fig 179

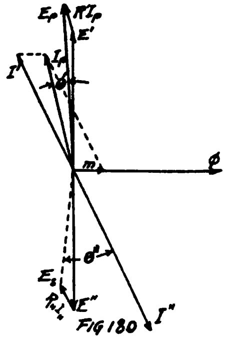


Fig 180

we can see the effect of coil resistance on the power factor. The main effect of coil resistance is on the regulation and efficiency. Fig. 178 has a poorer regulation than Figs. 179 and 180, since the $I_{,,}\ R_{,,}$ and the $I_{,}\ R_{,}$ are more nearly perpendicular in Fig. 178, thus producing more change in E_s and E_p. This means that with constant impressed primary voltage E_p the secondary voltage drops off badly as the load comes on. This is a bad feature, especially on a lighting transformer.

103. Effect of Magnetic Leakage.—In the equations:—

$$E' = 4.44\ \phi_{(max)}\ f.N.'$$

and

$$E'' = 4.44\ \phi_{(max)}\ f.N.''$$

if there is a leakage of flux then $\phi_{(max)}$ in the second equation is less than in the first equation. So that any leakage of the magnetizing flux or no-load flux tends to lower the secondary e.m.f. E″. This leakage amounts to very little.

In Fig. 176 the ampere turns N″ I″ produce a flux ϕ'' in phase with I″ (see Fig. 181, which is a part of Fig. 176). These ampere turns are opposed by equal and opposite ampere turns N′ I′ in the primary, which ampere turns produce a flux ϕ' in phase with I′

and equal and opposite to ϕ''. Because these two load fluxes, as they are called, oppose each other there is a leakage between the two coils (Fig. 182), and since the opposition increases with the load ($N'' I''$ increases with load), so will the leakage.

In Fig. 181 ϕ' and ϕ'' represent not the total flux that would be produced by $N' I'$ and $N'' I''$ respectively, but only the amount of each that leaks between the primary and secondary (two lines each in Fig. 182). The total fluxes ϕ' and ϕ'' do not exist, since they oppose, and hence annul, each other (if the ampere turns on the two spools of an Edison bipolar D.C. generator opposed each other, the fields would be unmagnetized and the flux passing into the armature would be zero).

Therefore in an actual transformer there are three fluxes:—

(1) The magnetizing or no-load flux, ϕ (Fig. 181)

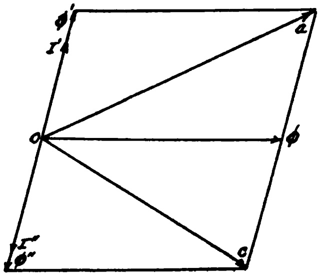


Fig 181

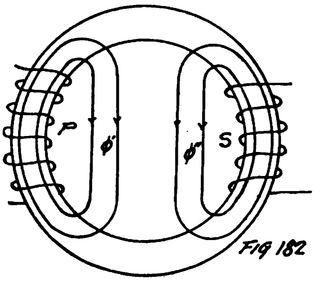


Fig 182

(2) The leakage of primary load flux ϕ' (Figs. 181 and 182)

(3) The leakage of secondary load flux ϕ'' (Figs. 181 and 182)

Evidently the total flux passing through the primary coil is the vector sum of ϕ and ϕ', or oa (Fig. 181), and the total flux passing through the secondary coil is the vector sum of ϕ and ϕ'', or oc.

Since any side of a triangle is the vector sum of the other sides we have

$$oa = oc + ac$$

or the total flux passing through the primary oa = the sum of the total flux passing through the secondary oc and the total leakage flux ac $(=\phi' + \phi'')$.

Now ϕ' varies as I'

and ϕ'' varies as I''

since $\frac{I'}{I''} = \frac{N''}{N'} = \text{a constant}$

therefore $\phi' + \phi'' = ac$ and varies as I'

or the total leakage flux in a transformer (ac) is in phase with I′ and is proportional to I′.

Since the flux in any induction coil is in phase with its current and proportional to its current, an induction coil could be so designed and placed in series with the primary that the flux produced by it would be in phase with its current I′ and equal to $\phi' + \phi'' = ac$.

Therefore a transformer with leakage could be entirely replaced by a transformer without leakage, but with an induction coil placed in series between the primary and the generator.

Fig. 183 represents a generator connected, in series with an induction coil (L), to a transformer; the transformer is without leakage and is connected to a pure resistance load (R).

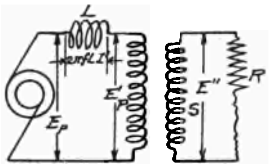


FIG 183

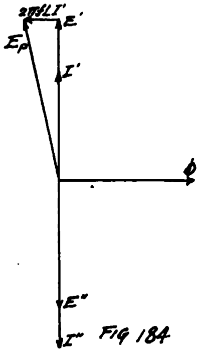


FIG 184

Fig. 184 is the clock diagram to correspond to Fig. 183. The magnetizing current and coil resistance (see Fig. 178) have been neglected.

E″ and I″ are in phase, and therefore E′ and I′ are in phase. The voltage to overcome the drop in the induction coil (L) is $2\pi fLI'$ and is 90° ahead of I′. So that the total voltage E_p is very little larger than E′, which means that leakage in a transformer with resistance load (lighting transformer) does not spoil the regulation.

By drawing figures similar to Fig. 184 for—

(*a*) highly inductive load and

(*b*) highly capacity load

we see that leakage causes the voltage to drop as the load comes on with inductive load (poor regulation), and may actually cause the voltage to rise as the load comes on with highly capacity load.

Leakage and coil resistance are the two main causes for poor regulation in transformers.

It is to prevent this leakage that the primary and secondary coils are sandwiched between each other. In building up a transformer core the laminations overlap. This reduces the reluctance of the iron core and reduces leakage and reduces m (Fig. 174). From a study of Fig. 174 we can see that the smaller m is, the better will be the power factor on the primary side, which, of course, is desirable.

104. To Find the Equivalent Resistance and Reactance of an Ideal Transformer.—Suppose a transformer connected to a load of resistance R and reactance X (Fig. 176).

If a simple circuit of resistance r and reactance x (when connected directly to the same supply circuit as the above transformer) takes the same current from the mains as does the primary of the transformer above, then this resistance r and reactance x have taken the place of the transformer and its load, and they are therefore called the equivalent primary resistance and reactance of the receiving circuit. Fig. 185 is similar to Fig. 176. Resolve E′ into its two components rI′ and xI′ in phase with and at right angles to I′ respectively, and in a similar manner resolve E″ into its two components RI″ and XI″.

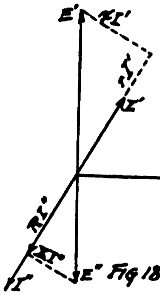


Fig 185

From similar triangles we get:—

$$\frac{xI'}{XI''}=\frac{E'}{E''} \qquad \text{but } \frac{E'}{E''}=\frac{N'}{N''}$$

therefore $$\frac{xI'}{XI''}=\frac{N'}{N''} \qquad \text{but } \frac{I'}{I''}=\frac{N''}{N'}$$

therefore $$\frac{x}{X}\frac{N''}{N'}=\frac{N'}{N''}$$

or $$x=X\left(\frac{N'}{N''}\right)^2 \qquad (66)$$

In exactly the same way $$r=R\left(\frac{N'}{N''}\right)^2 \qquad (67)$$

105. Losses in Transformers.—The losses in a transformer may be divided into two classes:—

(*a*) core losses (or iron losses).

(*b*) copper losses.

(*a*) The core losses consist of two parts:—

1. Hysteresis loss:—

$$W_h = a\ V\ f\ B^{1.6}_{(max)} \qquad (68)$$

where W_h = hysteresis loss in watts
V = volume of iron in cu. cm.
f = frequency
$B_{(max)}$ = the maximum flux density
a = hysteretic constant (about 3×10^{-10} for annealed wrought-iron).

2. Eddy current loss:—

$$W_e = b\ V\ l^2 f^2 B^2_{(max)} \qquad (69)$$

where W_e = eddy current loss in watts
V = volume of iron in cu. cm.
l = thickness of laminations in centimeters
f = frequency
$B_{(max)}$ = the maximum flux density
b = a constant depending on the specific electrical resistance (resistivity) of the iron and has a value of about 1.6×10^{-11}.

The formula for hysteresis is empirical and is due to Steinmetz. The formula for eddy currents can be derived from the relation:—

$$W = I^2R = \frac{E^2}{R}$$

(see Hay's "Alternating Currents," 1906, p. 107) the hysteresis loss is about 80%, and the eddy current loss about 20% of the iron losses.

(*b*) The copper losses consist of two parts:—

1. I''^2R'' in the secondary
2. I'^2R' in the primary.

106. Efficiency.—The efficiency of a transformer is the output divided by the input,

$$\text{or efficiency} = \frac{E''I''}{E''I'' + (W_e + W_h) + (I''^2R'' + I'^2R')} \qquad (70)$$

$$\text{or efficiency} = \frac{E'I' - (W_e + W_h) - (I''^2R'' + I'^2R')}{E'I'} \qquad (71)$$

The transformer is the most efficient piece of electrical apparatus and may have an efficiency of over 98½%.

107. All-day Efficiency.—When the secondary of a transformer is on open circuit and the primary is connected to the generator, evidently since there is the normal flux through the core (ϕ Figs. 174, 176, 177) there will be normal iron bases (W_e+W_h). Furthermore, since ϕ (Fig. 174) does not change as the load comes on, the core losses are constant at all loads, and are equal to the core losses with the secondary on open circuit.

ϕ (Fig. 174) is constant at all loads because in the expression

$$E_{,} = 4.44\ \phi_{(\max)}\ fN' \qquad \text{(See equation 63.)}$$

E', 4.44, f, and N' are all constants, and therefore $\phi_{(\max)}$ must also remain constant.

The copper losses are, of course, zero (neglecting m in the primary, Fig. 174) when the secondary is on open circuit, and they increase as the square of the current (I^2R).

The all-day efficiency is the ratio of the energy output to the energy input during the twenty-four hours. The usual conditions of practice are met in lighting transformers, if it is assumed that the transformer has full load for five hours and no load for nineteen hours. The core losses which continue twenty-four hours, and the copper losses which continue but five hours, should be so adjusted as to give a maximum all-day efficiency. A transformer which is loaded but a few hours per day should have relatively small core and large copper losses. Too large copper losses spoil the regulation. This applies to lighting transformers, in which it is very important to have good regulation. A power transformer is loaded, say, ten hours a day, and can therefore have more iron and less copper losses. The copper and iron losses are often made about equal at full load. The different proportioning of the iron and copper losses is one of the greatest points of difference in the design of lighting and power transformers.

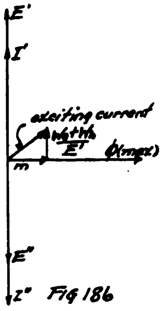


Fig 186

108. The Exciting Current.—The exciting current is the current that flows through the primary of a transformer when the secondary is on open circuit.

Fig. 186 is similar to Fig. 174. Since the magnetizing flux (ϕ) is many times larger than the leakage flux ($\phi'+\phi''$, Fig. 182), and since ϕ is the same at all loads, it follows that the hysteresis and the eddy current losses are normal and present at no load. Therefore if a wattmeter were placed in the primary side with the secondary on open circuit, it would read W_e+W_h (+ a very small copper loss due to the small current flowing in the primary, which loss is negligibly small).

In Fig. 186, since m is 90° from E′, the wattmeter would read zero if m were the only current flowing in the primary. But the wattmeter reads W_e+W_h, therefore there must be another component of the total no-load current in the primary, and it must be in phase with E′ to produce the wattmeter reading (core losses), and it is therefore equal to $\frac{W_e+W_h}{E'}$. m is called the magnetizing current (wattless component of the total no load current); $\frac{W_e+W_h}{E'}$ is the power component, and the vector sum of these two, or the total no-load current, is called the *exciting current.*

Power Component of Exciting Current (M_p):—

Let $M =$ exciting current

then $$M_p=\frac{W_e+W_h}{E'} \qquad (72)$$

Wattless Component of Exciting Current (M_w):—$\phi_{(max)}$ (Fig. 186) may be obtained from the equation:—

$$E'=4.44\ \phi_{(max)}\ fN' \qquad \text{(See equation 63.)}$$

then knowing the cross-section of the iron (a), $B_{(max)}$ may be obtained from the relation $B_{(max)}=\frac{\phi_{(max)}}{a}$

From a B-H curve obtain $H_{(max)}$ and

since $$H_{(max)}\ l=\frac{4}{10}\ \pi N'\ m_{(max)}$$

where $l=$length of magnetic circuit; $m_{(max)}$ may be obtained

or $$m_{(r.m.s.)}=M_w=\frac{10\ H_{(max)}\ l}{4\pi N'\sqrt{2}} \qquad (73)$$

109. Regulation.—When a transformer is on no load, the ratio of E_p to E_s is almost exactly the ratio of N′ to N″, but if the primary e.m.f. E_p is kept constant, the secondary e.m.f. (E_s) will decrease as the load comes on. The regulation is the rise in secondary voltage from full load to no load, divided by full load secondary voltage, with constant impressed primary voltage. Expressed in the form of an equation:—

$$\%\ \text{Regulation}=\frac{\text{no load }(E_s)-\text{full load }(E_s)}{\text{full load }(E_s)}\times 100 \qquad (74)$$

Example:—With a constant impressed primary voltage at all loads of 2200 volts, the secondary e.m.f. at no load is 112 volts and at full load 110 volts.

$$\% \text{ Regulation} = \frac{112-110}{110} \times 100 = 1.82\%$$

(For further information see regulation, General Electric bulletin, "Transformer Testing.")

110. Rating.—A transformer is generally rated by specifying the primary voltage, frequency, ratio of transformation, and power it can deliver at normal primary voltage and unity power factor.

111. Methods of Testing Transformers.—See General Electric bulletin No. 8418, "Transformer Testing."

112. Methods of Connecting up Transformers.—Transformers are generally always connected in parallel, and in the examples which follow there is assumed in each case a 1K.W. 10 to 1 step-down transformer (1000/100 = E′/E″).

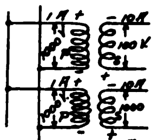


Fig 187

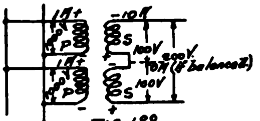


Fig 188

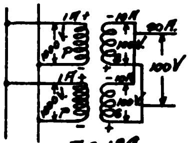


Fig 189

Fig. 187 shows two transformers connected in parallel on a single-phase circuit.

Fig. 188 shows two transformers, primaries in parallel, secondaries in series. By using the middle wire an ordinary three-wire system is obtained.

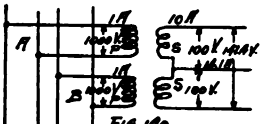


Fig 190

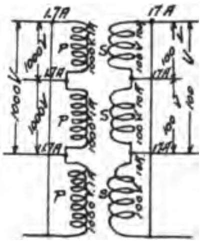


Fig 191

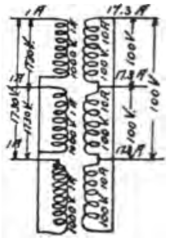


Fig 192

Fig. 189 shows two transformers, primaries in parallel, secondaries in parallel. When two transformers are used on two-phase four-wire mains, they are connected exactly as if they were two transformers connected on two entirely separate single-phase circuits and have no electrical connection with each other.

Fig. 190 shows two transformers connected for a two-phase three-wire transformation. (Note difference between voltages and currents in Fig. 190 and Fig. 188.) If the middle wire (Fig. 190) is removed, this becomes a two-phase to single-phase transformation.

Fig. 191 shows three transformers on a three-phase system, primaries connected △ and secondaries △. Since each transformer is 1 K.W. and $E' = 1000$ V., the current in each primary is 1 ampere $= i$, but in a △ $I = \sqrt{3}\, i$ therefore line current $= 1 \times \sqrt{3} =$ 1.7 amperes.

Fig. 192 shows three transformers on a three-phase system, primaries connected Y, secondaries △. Since in the primaries:—

$$e = 1000 \text{ volts}$$

and $$i = 1 \text{ ampere}$$

$$E = \sqrt{3}\, e = 1730 \text{ volts}$$

and $$I = i = 1 \text{ ampere}$$

In Fig. 191 the ratio of transformation of line voltages and currents is 10 to 1. In Fig. 192 the ratio of transformation of line voltages and currents is 17.3 to 1. These transformers could also be arranged primaries Y, secondaries Y, a 10 to 1 transformation on the line, or they could be arranged primaries △, secondaries Y, a 5.8 to 1 ratio of transformation on the line.

Three-phase generators are generally wound so that the voltage on any of the phases will be as small as possible.

Where a high ratio of transformation is required, transformers are connected primaries Y, secondaries △ (Fig. 192).

The △ method of connection is preferred to the Y because if one transformer should burn out, the system will continue to operate as a three-phase system, but can only give out .577 as much power as can the three transformers (see Franklin and Esty, "Alternating Currents," pp. 222, 223), while in a Y-connected system if one transformer is disabled the other two transformers will operate in series to supply a single-phase line.

These two conditions are more clearly understood by examining Fig. 193 (*a*) and (*b*). In Fig. 193 (*a*), if transformer C burns out at O, evidently the three lamps will still burn; but in Fig. 193 (*b*), if transformer C burns out at O, evidently the two lamps, x and z, will be an open circuit, and lamp y only will burn.

If a transformer has two primary coils and two secondary coils, by connecting these in series or in parallel three ratios of transformation may be obtained (see problem 166); but when connecting secondaries in parallel, one must be careful to have the two secondary e.m.f.'s opposing, as one secondary forms a short circuit on the other (just as in connecting two D.C. compound generators in parallel). When connecting primaries in series, or in parallel, one must be careful not to have the ampere turns produced by the one primary oppose the ampere turns of the other, for if they do the resultant ϕ will be zero, and since $E' = 4.44\ \phi_{(max)}\ f.N'.$, there can be no e.m.f. induced to oppose that from the generator.

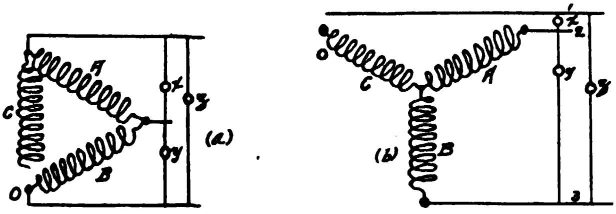


Fig 193

In either of the above wrong connections there is nothing but the resistance of the windings to oppose the flow of current, and serious damage to the transformer will result; it is thus very essential to determine the polarity of each primary and secondary coil. (See polarity test, General Electric bulletin, "Transformer Testing.")

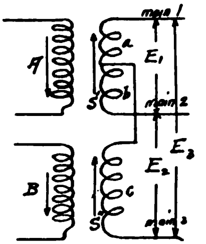


Fig 194

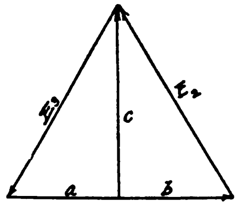


Fig 195

113. The Scott Transformer.—This transformer was devised to transform from two-phase to three-phase by Mr. Charles F. Scott, of the Westinghouse Electric and Manufacturing Company.

The general scheme of connections is shown in Fig. 194. There are two distinct transformers, A and B, with their primaries connected to the two-phase system and with each primary having the same number of turns N′.

From the middle point of one of the secondaries a lead is brought out and connected to the other secondary, as shown, and the three-phase circuit is taken off the mains 1, 2, and 3.

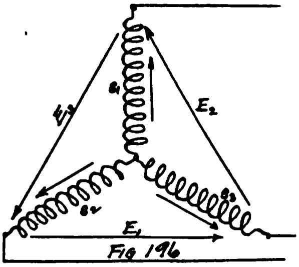


Fig 196

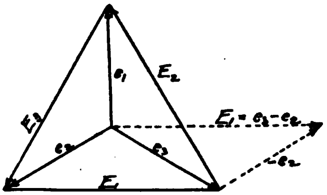


Fig 197

114. To Prove that the Scott Transformer Will Give a Perfect Three-phase System.—In Fig. 196 with the symmetrical circuit shown:—

$$E_{,} = e_3 - e_2$$
$$E_2 = e_{,} - e_3$$
$$E_3 = e_2 - e_{,}$$

but it is noticed in Fig. 197 that the $E_{,}$ obtained by subtracting vectorially e_2 from e_3 is equal and parallel to a line drawn from the end of e_2 to the end of e_3 and in the direction of e_3. Similarly E_2 and E_3 are obtained. Now, $E_{,}$, E_2, and E_3 are all equal and 120° apart. (Compare Fig. 197 with Fig. 115.)

In Fig. 195 a b represents the e.m.f. generated in S′ and c represents the e.m.f. generated in S″ 90° from S′ (primaries are connected to a two-phase circuit).

The line voltages $E_{,}$, E_2, and E_3 (Fig. 194) are shown vectorially in Fig. 195, and from the proof of Fig. 197 it is only necessary to choose the value of c such that Fig. 195 becomes an equilateral triangle in order that $E_{,}$, E_2, and E_3 may all be equal and 120° apart.

Suppose $E_{,} = E_2 = E_3 = 1$ then

by geometry $c = \sqrt{(E_2)^2 - (b)^2} = \sqrt{(1)^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} = .866$

So if $E_{,} = 1000 \text{ volts} = a + b = S'$

$c = 866 \text{ volts} = S''$

or if the number of turns of wire on S′ are 100 turns,
then the number of turns of wire on S″ are 86.6 turns.

Example:—On a certain Scott transformer the two similar primary coils each have 2000 turns and are connected to a 2200-volt two-phase circuit; if it is desired to supply three-phase mains at 2200 volts,

then evidently $(a+b) = 2200$ volts

and $$c = 2200 \times \frac{\sqrt{3}}{2} = 1905 \text{ volts}$$

or the number of turns on

$$S' \ (a+b) = N' \times \frac{2200}{2200} = 2000 \times 1 = 2000 \text{ turns}$$

and the number of turns on

$$S'' \ (c) = N' \times \frac{1905}{2200} = 2000 \times \frac{1905}{2200} = 1732 \text{ turns.}$$

If $I' = 10$ amperes

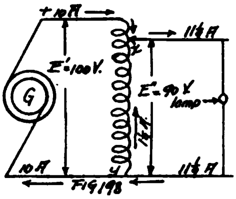
since $2\,E'I' = \sqrt{3}\,E''I''$

then $$I'' = \frac{2 \times 2200 \times 10}{\sqrt{3} \times 2200}$$

or $$I'' = \frac{20}{\sqrt{3}} = 11.55 \text{ amperes}$$

and since the secondary is Y-connected and the three-phase load is balanced, coils a, b, and c each carry 11.55 amperes. (For dephasing action in a Scott transformer see Franklin and Esty, "Alternating Currents," p. 229.)

115. **The Auto-transformer.**—The auto-transformer is one having but one winding, and is generally used where one or more voltages are necessary. It is made use of in starting squirrel-cage induction motors, and is generally used as a step-down transformer. Fig. 198 represents a step-down auto-transformer.



Example:—In Fig. 198 suppose the total number of turns on the coil is 100 and the primary voltage is 100 volts; if it is desired to reduce the voltage to 90 volts, then the secondary turns, or those between the points x and y, must be 90. Suppose, further, that the transformer receives 1 K.W. from the generator G;

then evidently $$I' = \frac{1000 \text{ watts}}{100 \text{ volts}} = 10 \text{ amperes (unity power factor).}$$

Assuming perfect efficiency $E''\,I'' = 1000$ watts

or $$I'' = \frac{1000}{90} = 11\tfrac{1}{9} \text{ amperes}$$

In Fig. 198 it is observed that $11\frac{1}{9}$ amperes flows from the lamp toward y, but only 10 amperes flows from y toward the generator; therefore $11\frac{1}{9}-10=1\frac{1}{9}$ amperes must flow *up* from y.

Energy transmitted $=1$ K.W.

Energy actually transformed = voltage across secondary × current in secondary

or energy actually transformed $=90\times1\frac{1}{9}=100$ watts.

Had an ordinary transformer been used, its capacity would have to be 1 K.W., but the auto-transformer of 100 watts or $\frac{1}{10}$ K.W. capacity can do the same work. Since only $\frac{1}{10}$ of the 1 K.W. has been actually transformed, it is called an auto-transformer.

116. The Constant Current Transformer.—This transformer has its primary connected to a constant potential circuit and delivers constant current from its secondary. It is chiefly used in connection with A.C. series arc lights and also with series tungstens for street lighting.

Fig. 199 represents the essential features of the constant current transformer. It is a shell type with long narrow windows; the primary coil is stationary and is placed below. The secondary is movable and is balanced by the weight W. It operates as follows: Suppose the secondary is connected to four arc lamps in series as shown. If one of these lamps is cut out (short-circuited), the resistance being reduced, the current in the secondary will increase. Since currents in the primary and secondary are 180° apart, they repel each other. This increase of secondary current will push S to the dotted position and increase the leakage of flux between the coils, which means less flux threads the secondary, and

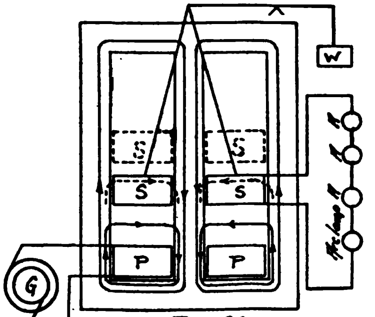


Fig 199

therefore the secondary e.m.f. falls and so the secondary current decreases to its normal value.

Again, if an additional lamp be added, the secondary current decreases, the repelling action is lessened, coil S comes closer to P, there is less leakage, therefore more secondary e.m.f., and hence the secondary current increases again to its normal value. (For illustrations, see Westinghouse bulletin 1084, "The Westinghouse Series A.C. Arc-lighting System," Figs. 10 to 13.)

PROBLEMS.

146. A 10 to 1 step-down transformer has 1000 turns on its primary and is connected to a 10,000-volt generator. Find E'' and N''.
147. The transformer of problem 146 delivers 100 K.W. to a pure resistance receiving circuit. Assuming perfect efficiency, find I'' and I'.
148. A transformer having 100 turns of wire on its primary receives from a generator 1000 amperes at 550 volts, and steps this energy up to 22,000 volts. Allowing 800 circular mils of wire in primary and secondary for each ampere of current, find N'' and size of wire in primary and secondary. *Ans.*:—$N''=4000$; $P=800{,}000$ circular mils; $S=20{,}000$ circular mils.
149. The iron in the transformer in problem 148 has a sectional area of 70 sq. inches. With a frequency of sixty cycles determine the maximum flux, $\phi_{(max)}$; and the maximum flux density, $B_{(max)}$. *Ans.*:—$\phi_{(max)}=2{,}063{,}000$. $B_{(max)}=29{,}470$ lines per sq. inch.
150. A certain core type transformer has 600 turns on its secondary, 200 mils in diameter; the primary wire is one-tenth as large in area. The sectional area of the iron core is 10 sq. inches and the maximum flux density in the core is 5000 lines per sq. cm. At twenty-five cycles find the primary and secondary e.m.f.'s. *Ans.*:—$E''=214.8$ volts; $E'=2148$ volts.
151. Allowing 800 circular mils per ampere in problem 150, what are the primary and secondary currents? What is the rating of the transformer in K.W.? *Ans.*:—$I''=5$ amperes; $I'=50$ amperes; K.W. output = 107.4 K.W.
152. A 10 to 1 step-down transformer delivers 500 amperes at 5000 volts to a receiving circuit of which the power factor is .8. Find the equivalent primary resistance and reactance of this receiving circuit. *Ans.*:—$r=800$ ohms, $x=600$ ohms.
153. A transformer having 120 turns on its primary and 1200 turns on its secondary is connected to a 110-volt generator through a non-inductive resistance of 1.1 ohms. The secondary is connected to a pure resistance of 11 ohms. Find (*a*) E''; (*b*) E'; (*c*) I''; (*d*) I'; (*e*) power delivered by the secondary; (*f*) power given out by the generator. Also make a diagram of connections and a clock diagram showing all the voltage and current relations. *Ans.*:—(*a*) 999.9; (*b*) 99.99; (*c*) .909; (*d*) 9.09; (*e*) 909 – watts; (*f*) 999.9 watts.
154. In the transformer of problem 153 a secondary e.m.f. of 550 volts is impressed on a purely inductive receiving circuit whose reactance is 110 ohms; find (*a*) E'; (*b*) I''; (*c*) I'; (*d*) voltage at the generator. *Ans.*:—(*a*) 55; (*b*) 5; (*c*) 50; (*d*) 77.5 volts.
155. A 100-K.W. 1000/100 volt transformer has a primary resistance $R'=.05$ ohm, a secondary resistance $R''=.0005$ ohm, and core losses (W_e+W_h) of 1000 watts. Calculate the ¼, ½, ¾, full, and 1¼ overload efficiencies and plot the efficiency curve. *Ans.*:—Full load, 98%.
156. Assuming the transformer in problem 155 to be operating at full load five

hours each day and idle nineteen hours, calculate the all-day efficiency. *Ans.:*—94.5%.

157. In problem 155 what would the impressed voltage (E_p, Fig. 178) on the primary be at full load if the secondary terminal voltage (E_s) is to be 100 volts at full load? Calculate the regulation. Neglect exciting current and leakage. *Ans.:*—1010 volts; regulation = 1%.

158. A given transformer is rated at 10 K.W. 2200 volts primary and twenty-five cycles. With these rated conditions we will assume W_e, W_h, and $W_{(I^2R)}$ normal.
    (*a*) If the transformer delivers 12 K.W. at rated e.m.f. and frequency, find $W_{(I^2R)}$ in terms of normal.
    (*b*) Suppose the transformer is used on a sixty-cycle circuit, but with normal voltage; find W_e and W_h in terms of normal.
    (*c*) Suppose the transformer is used on a 3000-volt circuit, but with normal frequency; find W_e and W_h in terms of normal.
    (*d*) With E′ at 3000, what load will give normal W_c?
    Ans.:—(*a*) 1.44 × normal $W_{(I^2R)}$; (*b*) no change in W_e; .59 × normal W_h; (*c*) 1.86 × normal W_e, 1.64 × normal W_h; (*d*) 13.6 K.W.

159. Make a clock diagram showing the effect of leakage on a transformer, (*a*) R. and L. load on secondary, (*b*) R. and C. load on secondary. (See Fig. 184.)

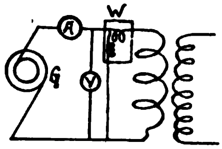
160. Make a clock diagram showing the effect of coil resistance and leakage on a transformer, (*a*) pure resistance (R) load, (*b*) R. and L. load, (*c*) R. and C. load.
    Note:—Combine Figs. 178 and 184, neglecting exciting current.

161. The resistances of the primary and secondary of a transformer are R′ = 1 ohm, R″ = .01 ohm. The equivalent leakage reactances of the primary and secondary are .7 ohm and 3.3 ohms. This is a 10 to 1 step-down transformer with a pure resistance load of 5 ohms; E″ is 100 volts. Neglecting exciting current, calculate E′ and the regulation. *Ans.:*—E′ = 1004.03 volts; regulation = 4.03%.

162. In problem 161 calculate E′ and regulation with all conditions unchanged except that the p.f. of the load on the secondary is .8. *Ans.:*—E′ = 1008.00008; regulation = 8.00008%.

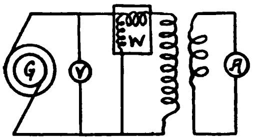
163. On a core-loss test on a certain transformer the following readings were taken:—

A = 3 amperes
W = 220 watts
V = 110 volts



Calculate no-load power factor. With pure resistance load on the secondary, calculate the power factor on the primary, (*a*) at half load, (*b*) at full load. *Ans.:*—No-load power factor = .667; half-load p.f. = .9967; full-load p.f. = .998.

164. In the transformer of the preceding problem on an impedance test the following readings were taken:—



A = 50 amperes
W = 100 watts
V = 50 volts

By the voltmeter-ammeter method the resistance of the coils was found to be R′ = .02 ohm and R″ = 2 ohms. Calculate the full-load efficiency. *Ans.:*—94.5%.

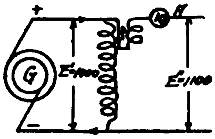
165. From the data given in the two preceding problems calculate the regulation in % (*a*) for unity power factor full-load current, (*b*) for .6 power factor full-load current, (*c*) for .6 power factor half-load current.

166. A certain 1 K.W. 10 to 1 step-up transformer has two primaries and two secondaries and is connected to a 100-volt generator. Find the secondary phase and line voltages and currents:—
   (*a*) primaries connected in parallel, secondaries in parallel;
   (*b*) primaries connected in series, secondaries in series;
   (*c*) primaries connected in series, secondaries in parallel;
   (*d*) primaries connected in parallel, secondaries in series.

Does case (*a*) or (*b*) have the greater copper losses? What is the advantage of (*c*) over (*d*) and vice versa? Assuming the core losses in case (*a*) normal, compare cases (*b*), (*c*), and (*d*) with case (*a*). Make a figure for each case, similar to Fig. 187, and put the values on the figure.

167. Assuming the primary line voltage from a three-phase generator to be 1000 volts and the power delivered by three 10 to 1 step-down transformers to be 17320 watts. Find the secondary phase and line voltages and currents:—
   (*a*) Primaries connected △, secondaries Y;
   (*b*) Primaries connected Y, secondaries Y.

168. A 1 K.W. 1000/100 volt transformer is connected as an auto-transformer with its secondary in series with the line as shown. As connected it delivers 1100 volts and 10 amperes:—



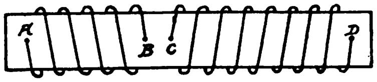
   (*a*) Find the current in each coil of the transformer.
   (*b*) Which coil is receiving power, and therefore the primary coil, and how much?
   (*c*) Which coil is delivering power, and therefore the secondary coil, and how much?

169. Reverse the connections of the secondary in the figure, problem 168, and assume that it still delivers 10 amperes:—
   (*a*) Find E'' and the current in each coil of the transformer;
   (*b*) Which coil is receiving power, therefore the primary, and how much?
   (*c*) Which coil is delivering power, therefore the secondary, and how much?

170. How much power is actually transmitted to the receiving circuit in (*a*) problem 168, (*b*) problem 169. (*c*) How much does this power exceed the rating of the transformer in each case? *Ans.*:—11 K.W.; (*b*) 9 K.W.

171. How must Fig. 198 be arranged so that 1 K.W. may be delivered to the lamps at 110 volts?
   (*a*) Find the ratio of N' to N''.
   (*b*) Find the current flowing in each coil of the transformer.

172. In the figure shown the coil C D is connected to an 800-volt generator. Show how the coils must be connected together to obtain:—
   (*a*) step-up transformation;
   (*b*) step-down transformation.



173. With the coils (problem 172) arranged for step-down transformation the service mains deliver current to a receiving circuit of 30 ohms:—
   (*a*) Does coil A B deliver or receive power, and how much?
   (*b*) Does coil C D deliver or receive power, and how much?
   (*c*) What is the total power delivered?
   (*d*) What is the current flowing in each part of the circuit?

174. A 200/8000-volt step-up transformer is connected to phase A of a two-phase system and a 200/6000-volt step-up transformer is connected

to phase B; find the value and phase of the secondary e.m.f. if the two secondary coils are connected in series. *Ans.:*—10,000 volts 36° 54′ from one secondary e.m.f. and 53° 6′ from the other secondary e.m.f.

175. Two transformers have their primaries connected to a two-phase 500-volt circuit. Each primary has 500 turns of wire. The secondaries are connected in series to produce a single-phase circuit. It is desired that this single-phase circuit shall have an e.m.f. of 200 volts 60° ahead of one of the two secondary e.m.f.'s and 30° behind the other secondary e.m.f. Find the number of turns in each secondary. *Ans.:*—173.2 turns; 100 turns.

176. A certain Scott transformer is to transform from 10,000 volts two-phase to 1000 volts three-phase. If each primary, two-phase side has 2000 turns, find the number of turns on each secondary. *Ans.:*—(*a*) 200 turns on a+b (Fig. 194); (*b*) 173.2 turns on c.

177. In a sixty-cycle Scott transformer the area of the iron core is 75 sq. cm. and $B_{(\max)} = 3500$ lines per sq. cm. It is to transform 11 K.W. at 1100 volts two-phase to 110 volts three-phase; find:—

(*a*) The turns on each primary and secondary coil, and

(*b*) the current flowing in each.

(*c*) Does the relation $\frac{I'}{I''} = \frac{N''}{N'}$ hold between each primary and the corresponding secondary?

Ans.:—(*a*) N′ = 1573 (a+b) = 157.3; *c* = 136.1; (*b*) current in each two-phase coil = 5 amperes; (*c*) current in a = b = c = 57.8 amperes.

CHAPTER XI.

POLYPHASE INDUCTION MOTORS.

117. Use.—The synchronous motor is not self-starting, has to be synchronized, and can run at only one speed. By far the greater amount of service demands a motor that is self-starting, can have one or more speeds, and produces a large starting torque. For this large class of work the polyphase induction motor has been designed. It is still a question whether the induction motor is a very satisfactory variable speed motor; induction motors are inherently very nearly constant speed at all loads.

118. Construction.—As its name implies, it is, like the transformer, an induction machine, there being no electrical connection between the stationary and moving parts. The stationary elements is called the *stator*, and is connected directly to the line and is entirely similar to the primary of the transformer. The moving element is called the *rotor*, and has no electrical connection with the stator, but is short-circuited on itself, and therefore is similar to the secondary of the transformer. The secondary of a transformer is stationary and has its axis parallel to the axis of the primary, and can therefore have no torque exerted on it by the primary. The rotor (or secondary) of the induction motor revolves on an axis, and this axis is at right angles to the axis of the stator (or primary), and does therefore have a torque exerted on it by the stator.

Polyphase induction motors are built for two-phase and three-phase circuits. The stator winding is exactly similar in principle to the armature winding on a polyphase generator.

Fig. 200 represents one method of arranging the stator windings for a four-pole two-phase induction motor. The radial lines xx represent slots on the stator parallel to the rotor shaft; aa and bb are the end connections. Phase B is not drawn, for clearness' sake. Phase A produces the four poles, NSNS, as shown, and likewise phase B will produce four poles midway between those of phase A; therefore it is called a four-pole motor.

Fig. 201 shows one of the three windings of a four-pole three-phase stator.

There are two kinds of rotors:—

(*a*) A *wound rotor*, which is exactly similar to the stator winding and is almost always three-phase Y-connected for both two- and three-phase motors.

(*b*) A *squirrel-cage rotor*, which consists of a large number of copper bars equally spaced in slots parallel to the shaft, and all of these bars are bolted at each end to a copper ring. This, of course, makes a circuit of very low resistance. The stator and rotor are both laminated to reduce the iron losses. (See the Crocker-Wheeler, General Electric, and Westinghouse bulletins on the induction motor.)

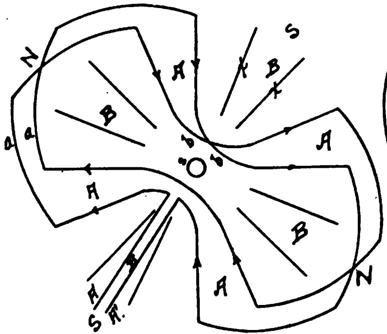


Fig 200

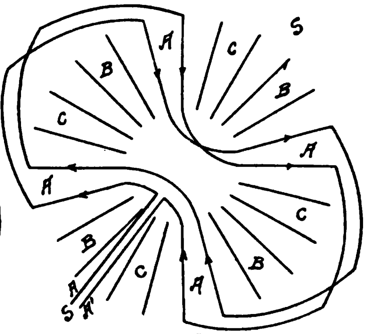
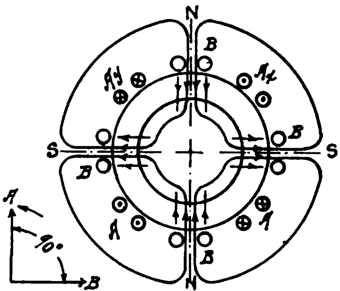


Fig 201

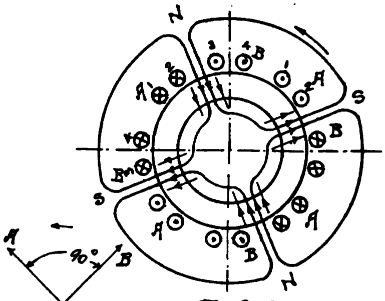
119. The Production of a Rotating Field.—Figs. 202 and 203 are supposed to be a section at right angles to the shaft through the stator and rotor of Fig. 200. The A and B slots on the stator are bunched together to make the figures clearer and the rotor conductors are not drawn in.

In Fig. 202 (*a*) the current in phase A has just reached the positive maximum value and the current in phase B is then zero (90° apart) (Fig. 202, b). We will assume, further, that on Fig. 202 (a) the current in phase A is positive when flowing toward the reader at x and away from the reader at y. This produces a flux *into* the rotor at NN and *out of* the rotor at SS, or produces the four poles, NSNS, in the positions shown. Current in phase B is now zero, and therefore B produces no flux.

In Fig. 203 (a) phase A is still positive, but decreasing, and phase B is no longer zero, but is positive and increasing (Fig. 203, b). The four turns, 1, 2, 3, 4, tend to produce an N-pole, not vertically upward, as in Fig. 202 (a), but one-sixteenth of a revolution to the left, and it will be noticed that all the four poles produced in Fig. 203 (a) are shifted one-sixteenth of a revolution in the direction of the rotating arrow from those in Fig. 202. On a four-pole machine one revolution $=2\times360°=720$ electrical degrees, and one-sixteenth of a revolution $=\frac{1}{16}$ of $720=45$ electrical degrees; and it is noticed that Fig. 203 (b) is 45° from Fig. 202 (b). In other words, in two complete cycles the four poles will have made one complete revolution in a clockwise direction. It is only the field (ϕ) that moves; the stator, of course, being stationary.



(b) Fig 202 (a)



(b) Fig 203 (a)

By using other figures similar to the two drawn it can be proved that if a two-phase alternating current is supplied to the stator of an induction motor, a rotating field will be produced. The speed of this rotating field is a pair of poles in one cycle. In Fig. 203 it is seen that the field will make one revolution in two cycles, or with sixty cycles supplied it will make 30 r.p.s. Similarly, on a two-pole sixty-cycle motor the field will rotate 60 r.p.s., and on a six-pole sixty-cycle motor the field will rotate 20 r.p.s.

The instantaneous value of the flux produced

by phase A $\phi'=\phi'_{(max)}\sin 90°$ } —Fig. 202 (b)
and by phase B $\phi''=\phi''_{(max)}\cos 90°$ }

by phase A $\phi'=\phi'_{(max)}\sin 45°$ } —Fig. 203 (b)
and by phase B $\phi''=\phi''_{(max)}\cos 45°$ }

therefore total field $=\sqrt{(\phi')^2+(\phi'')^2}=\sqrt{(\phi'_{(max)}\sin 45°)^2+(\phi''_{(max)}\cos 45°)^2}$

but $\phi'_{(max)}=\phi''_{(max)}$ (both phases are similarly wound)

therefore total field $=\phi_{(max)}\ [\sqrt{(\sin^2 45°+\cos^2 45°)}]$

but $(\sin^2 45°+\cos^2 45°)=1$

$\therefore$ total field $=\phi_{(max)}$, and is therefore constant in magnitude.

In the same way a three-phase induction motor produces a rotating field.

This magnetic field is exactly the same as would be produced by revolving the fields of a four-pole D.C. machine.

120. To Prove that an Induction Motor Will Produce a Continuous Torque.—(*Note:*—For the sake of clearness and simplicity a two-pole motor has been assumed in the following discussions.) Since the rotating field of a two-pole motor is exactly the same as rotating the fields of a two-pole D.C. machine, let Fig. 204 represent a two-pole D.C. machine with its field rotating counter-clockwise, as indicated by the circular arrow.

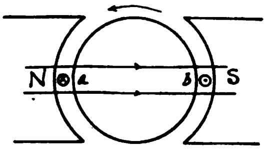


Fig 204

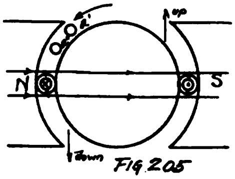


Fig 205

Suppose the armature is stationary; then voltage will be induced, in the conductor a on the armature, away from the reader and in b towards the reader.

Since current is flowing away from the reader in a, this current will produce a circular line of force around a, as shown in Fig. 205 (this is armature flux). But it is seen that this armature flux adds to the field flux above the conductor and opposes the field flux below the conductor. Since the fields push on the conductor and the stronger field is above, evidently conductor a will be pushed *down*, and in the same way conductor b will be pushed *up*. In other words, the push on the two conductors tends to produce rotation counter-clockwise.

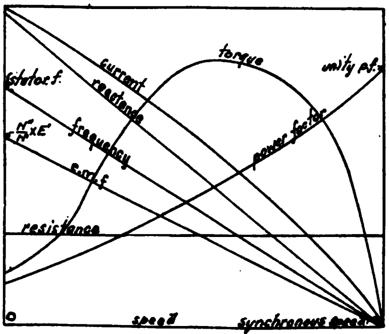


Fig 206

Therefore the rotor of a polyphase motor will rotate in the same direction as the rotating field.

121. The Action of the Motor for Varying Speeds.—1. *Rotor frequency:*—At rest the rotor frequency = stator frequency, but since the rotor follows the rotating field, the frequency decreases as the motor speeds up, and is zero at synchronous speed, which is the speed of the rotating field. (See Fig. 206.)

2. *Rotor resistance:*—Constant at all speeds.

3. *Rotor e.m.f.:*—Is a maximum at O speed because the rotating field is being cut at a maximum rate at that speed and decreases uniformly to O value at synchronous speed.

4. *Rotor reactance:*—Rotor reactance $X = 2\pi fL$, and decreases with f, since 2, π, and L are constant.

5. *Rotor current:*—Rotor current $I_r = \frac{E_r}{\sqrt{R_r^2 + X_r^2}}$. In this equation the numerator decreases more rapidly than the denominator, since R remains constant; therefore the curve is not a straight line (see Fig. 206). At synchronous speed $I_r = \frac{O}{\sqrt{R_r^2 + O}} = O$.

6. *Rotor power factor:*—Power factor $= \frac{R_r}{\sqrt{R_r^2 + X_r^2}}$, and since the numerator is constant and the denominator decreases, p.f. will increase, but not according to a straight line. At synchronous speed p.f. $= \frac{R_r}{\sqrt{R_r^2 + O}} = 1$.

7. *Torque:*—We have always learned that torque varies as field flux × armature flux or $\alpha\ \phi \times I_r$. Now, if the rotating field ϕ is constant, then torque αI_r (Fig. 206), which would make the torque a maximum at start, which is desirable. Torque is not a maximum at start for two reasons:

First: At start, owing to the heavy current in the rotor (Fig. 206), a great deal of the stator flux leaks across the air-gap between the stator and the rotor, just as leakage in a transformer increases with I'' (in this case I_r). But in the case of the transformer this leakage is much smaller, because the magnetic circuit has no air-gap with its high reluctance. Therefore the useful stator flux to produce torque is smaller at start than later, which reduces starting torque.

Second: At start the rotor frequency is high, therefore the power factor is low, and therefore the rotor current lags behind the rotor e.m.f. a maximum amount at start. In Fig. 205 e.m.f. is a maximum when armature coil is in strongest field (or position a); since current, and hence armature flux (circular line in Fig. 205), lags behind e.m.f., coil a will be in the position a′ when current and armature flux are a maximum; but a′ is not as strong a part of the rotating field as a, therefore the torque will be reduced, and the speed torque curve will actually have some such shape as shown by the full line curve in Fig. 207.

122. The Effect of Inserting Resistance in the Rotor Circuit.—If rotor resistance be increased, I_r will be reduced; this reduces leakage. An increase of R_r improves power factor $\left(=\frac{R_r}{\sqrt{R_r^2+X_r^2}}\right)$ and allows armature current and flux to react more nearly in phase with E_r (position a″, Fig. 205), which means in a stronger rotating field. For these two reasons an increase in rotor resistance will produce a larger starting torque (desirable) and a decrease in rotor current (desirable). This method is therefore frequently employed for starting induction motors. Fig. 207 shows a set of speed-torque curves for different rotor resistances. These torque curves all pass through zero at synchronous speed, since at that speed I_r is zero, and therefore there is no armature flux to react on the rotating field ϕ.

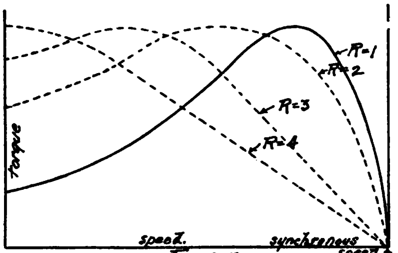


Fig 207

Since the motor has windage and friction when running at synchronous speed, and since the torque at that speed is zero, no induction motor can run as fast as synchronous speed.

Induction motors with resistance in the rotor circuit can now be made to start with full-load torque and a current not exceeding full-load current by 25%, therefore not appreciably lowering the line voltage (curve R=4, Fig. 207).

Because of the heavy current and poor power factor at start (inductive load on the generator), the starting of a large induction motor will reduce the line voltage. (See armature reactions, art. 73.)

123. Slip.—Slip is equal to the speed of the stator flux (n) minus the speed of the rotor flux (n′) divided by the speed of stator flux; or, expressed in percentage and in the form of an equation:—

$$\% \text{ slip } (s) = \frac{n - n'}{n} \times 100. \tag{75}$$

at synchronous speed $n' = n$ and $\%$ slip $= 0$

at zero speed $n' = 0$ and $\%$ slip $= \frac{n}{n} \times 100 = 100\%$.

The slip on large induction motors from no load to full load does not amount to over 1½% to 2%, so they are practically constant speed motors.

124. Methods of Starting.—There are two general methods of starting most commonly used in this country.

1. *Resistance placed in series with the rotor circuits.* Obviously this method is only applicable to motors with wound rotors (called slip ring induction motors), because a squirrel-cage rotor has no definite paths for the current. (See art. 135 and Fig. 216.)

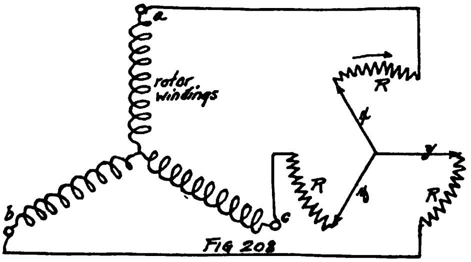


Fig 208

Fig. 208 shows a diagram of connections for starting a motor with a three-phase Y-connected rotor. The three slip rings are at the points a, b, and c, and the starting resistances, R, R, R, are external to the motor, necessitating three slip rings and brushes. The three arms, x, y, and z, are electrically connected together, and as the motor speeds up the arms are turned in the direction of the arrow, cutting out resistance. We have seen how this resistance will cut down the current and increase the torque at starting.

2. *Low voltage on the stator at start.* This method is always used with motors with squirrel-cage rotors.

Theory:—The current in the rotor varies directly with the primary voltage. For at start if the primary voltage is halved, the rotating field ϕ is halved, and therefore the e.m.f. in the rotor is halved and also the rotor current.

Torque depends on rotating field ϕ and rotor current, and since both have been halved, the torque has been reduced four times, or torque varies approximately as E^2 impressed on the stator.

In this method an autotransformer is used in each phase and the device is called a *starting compensator* or *auto-starter.*

Fig. 209 shows the method of reducing the voltage one-half. With 110 volts on the stator at start, it is evident that the line current will only be half the current taken by the stator. When the motor has nearly reached constant speed, the contacts xx are quickly moved to the running points yy, giving the stator the full 220 volts.

Fig. 210 shows the connections for a General Electric three-phase auto-starter. There are four taps from each secondary, and for very light loads the low voltage taps 1 may be used, and for heavy loads taps 2 or 3, etc. The fuses F are not in the starting circuit, owing to the large currents at start, but are placed in the running circuit. Small motors up to 5 H.P. are generally thrown directly on the line without any starting device.

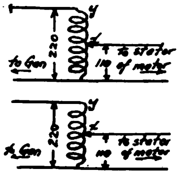


FIG 209

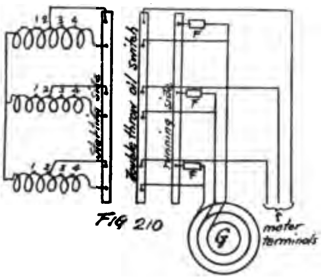


FIG 210

125. The Similarity between an Induction Motor and a Transformer.—When an induction motor is at rest, the rotor is exactly like the secondary of a transformer on short circuit.

When an induction motor is running at synchronous speed ($n' = n$), then there is no flux cutting the rotor, and the rotor current is zero, which is exactly the condition in a transformer with its secondary on open circuit. At synchronous speed, therefore, the only current flowing in the stator is that required to produce the rotating flux or magnetization, and the current to produce the core losses (W_h and W_e) in the stator, which, as in the transformer, is called the exciting current. (See Fig. 186.)

When the machine is loaded, the speed drops and the slip increases; this increases E_r (E'') and I_r (I''), which produces a demagnetizing action on the stator; this action, as in the case of the transformer, is neutralized by the appearance of an opposite current in the primary (load current I') (see Fig. 174). The more the slip, the larger I_r (I''), and in the case of a transformer a decrease of resistance will increase I''. Therefore any increase of load on an induction motor is similar to decreasing the secondary resistance on a transformer.

When an induction motor is running with a given slip (say, 50%), if the frequency of the stator e.m.f. and current is f (say, 60), the frequency of the rotor e.m.f. and current is $s \times f$; in this case thirty cycles. But the reactions of the rotor flux and rotor current on the stator are of the same frequency as the stator e.m.f. (60); because the flux produced by the secondary revolves relative to the secondary with a speed equal to the frequency of the secondary e.m.f., and the speed of this flux (30) plus the mechanical speed of the secondary (30) gives this flux the same speed relative to the stator (30+30) as the speed of the rotating field produced by the stator. For this reason the secondary of an induction motor has the same frequency relative to the primary as has the secondary of a transformer.

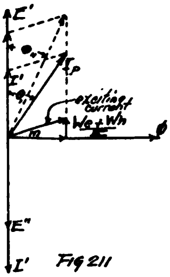


Fig 211

126. The Characteristic Curves of an Induction Motor.

126. The Characteristic Curves of an Induction Motor.—The characteristic curves of all motors are generally plotted with H.P. output as abscissas.

Fig. 211 represents the phase relations in an induction motor, and is entirely similar to Fig. 186. I'' is drawn in phase with E'', because the slip, and hence the rotor frequency, is so small it may be neglected, which means that the power factor in the rotor is unity, the rotor reactance is negligibly small, and, since W_e and W_h in the rotor depend on f, they are both negligibly small.

There is one important difference between Figs. 186 and 211. In Fig. 211 m is much longer than in Fig. 186, because of the increased reluctance due to the air-gap. This, of course, means that the induction motor has a larger exciting (no-load) current and a poorer power factor at no load and more leakage than the transformer. For these reasons the air-gap in induction motors is made as small as possible.

1. *Power factor curve:*—Referring to Fig. 211, the p.f. is not zero at no load because of the vertical component (W_e+W_h) of exciting current. As the load increases the load current I′, which is in phase with E′, increases, and thus the angle between E′ and I′ decreases from θ' to θ'', which means p.f. ($\cos \theta$) increases.

2. *Current curve:*—As in a transformer (I″ and therefore),

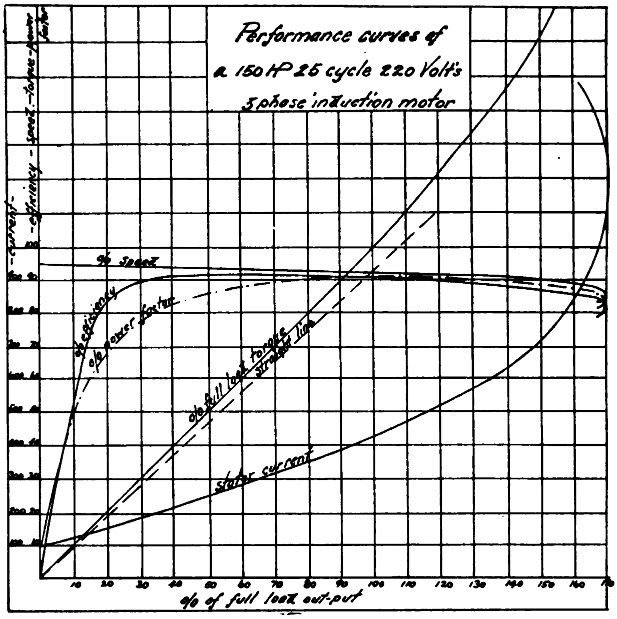


Fig. 212.

I_p increases with increase of load, but does not start from zero because of the exciting current which is present at no load.

3. *Speed curve:*—Speed drops off as load comes on to give a higher rate of cutting, thus increasing E″ and I″.

4. *Torque curve:*—H.P.$=2\ \pi\ n'\ T=$output of any rotating shaft. If n′ remained constant, then T would vary directly with H.P., or would be a straight line, but since n′ decreases slightly, the torque curve will have to increase a little faster than a straight line.

5. *Efficiency curve:*—

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input—losses}}{\text{input}}$$

$$= \frac{E'I_p \cos\theta - (I_p{}^2R' + I''^2R'') - [(W_e + W_h) \text{ in stator}]}{E'I_p \cos\theta} \qquad (76)$$

Since these losses are similar to those of any other electrical machine, the curve will be similar in shape to the efficiency curve of any other motor or generator.

The efficiency of a 20 H.P. two-phase induction motor is about 86% at half load, 87% at three-quarter load, 87% at full load.

127. Methods of Speed Control.—Since the slip on induction motors is so small, they are practically constant speed motors, but since so much service demands variable speed, several methods of varying the speed have been devised, two of which are most commonly used.

First method:—By varying the number of poles on the stator. In the beginning of this chapter we learned that reducing the number of poles increased the speed in like proportion, so that if the stator winding of a sixteen-pole motor could be quickly changed by a suitable switching device to make an eight-pole motor, the speed would be about doubled. The objection to this method is that it greatly complicates the stator winding.

Second method:—By inserting resistance in the rotor circuit—exactly the same as the first method given for starting (see Fig. 208). This method, of course, cannot be applied to motors with squirrel-cage rotors, and therefore squirrel-cage motors are not variable speed motors.

Theory of second method: With full load and no resistance in the rotor circuit the slip is, say, 2%; and it will vary from no load to full load. If resistance be inserted in the rotor circuit, the slip will have to be increased to produce more rotor e.m.f. to keep the rotor current up to its proper value; in other words, the motor will slow up. Therefore any increase in rotor resistance will decrease the speed.

There are two objections to this method:—

First:—The speed curve is so steep (H.P. output as abscissas) that if a certain resistance is inserted in the rotor to give a certain reduced speed, this reduced speed is constantly changing over a wide range for each small change in load.

Second:—Any increase in rotor resistance will, of course, lower the efficiency. The second method is more generally used.

128. Two-phase Versus Three-phase Induction Motors.—Three-phase motors, because of the better distribution of the stator windings, have a slightly higher efficiency than two-phase motors. The power factor is also somewhat better, especially at light loads; the copper loss in the line is somewhat less. These reasons recommend three-phase motors before two-phase motors for new plants. In large cities, where the power is generally bought, the motor must, of course, be of the same phase as the supply, and even in a private plant it is better, so that in case of emergency the motors can be temporarily connected with the local supply.

129. Relation between Speed and Efficiency.—Of the total power supplied to the stator a part is used in overcoming stator resistance and a part is used in overcoming stator iron losses (W_e+W_h), and the remainder is supplied to the rotor. If the torque exerted on the rotor by the rotating field (ϕ) is T, then the power delivered to the rotor is 2π n T. If the rotor had no copper losses and no windage and friction losses (windage and friction losses are neglected), it would have a speed n, and the mechanical output would be 2π n T, but because of rotor resistance the mechanical output of the rotor is only 2π n′ T. With the above assumptions:—

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{2\pi\, n'\, T}{2\pi\, n\, T} = \frac{n'}{n} \tag{77}$$

Calling W the electrical input to the stator, and W_m the mechanical output of the rotor. Since W_m is less than W (2π n′ T is less than 2π n T), evidently 2π n T $-$ 2π n′ T is the electrical power developed in the rotor windings (W_e); hence we have the proportion:—

$$\begin{aligned} &W : W_m : W_e = 2\pi\, n\, T : 2\pi\, n'\, T : (2\pi\, n\, T - 2\pi\, n'\, T) \\ \text{or } &W : W_m : W_e = n : n' : n - n' \end{aligned} \tag{78}$$

Analogy: A shaft A, with speed n, drives a shaft B at a slower speed, n′, through a friction clutch.

Power delivered by shaft A is 2π n T (W)
Power delivered to shaft B is 2π n′ T (W_m)
Power lost as heat due to the slipping of the clutch = 2π n T $-$ 2π n′ T ($I_r^2R_r$).

$$\text{Efficiency of transmission} = \frac{2\pi\, n'\, T}{2\pi\, n\, T} = \frac{n'}{n}$$

130. The Induction Motor Used as a Frequency Changer.—When an induction motor is run at synchronous speed, $n' = n$, and the electrical power generated in the rotor W_e is zero and frequency in rotor is zero. As motor slows up n' decreases, $W_e = 2\pi\, n\, T - 2\pi\, n'\, T$ increases and so does rotor frequency.

At rest $n' = o$, $W_e = 2\pi\, n\, T - o =$ electrical power supplied to stator, and rotor frequency = stator frequency. Now, if the relative speed between stator and rotor is still further increased by driving the rotor backward at speed n', then evidently

$$\begin{aligned} W_e &= 2\pi\, n\, T + 2\pi\, n'\, T \\ W &= 2\pi\, n\, T \\ W_m &= 2\pi\, n'\, T \end{aligned}$$

But since the motor is driven backward it is no longer giving out mechanical power W_m, but receiving it.

Therefore $$W_e = W + W_m$$
and $$W_e : W : W_m = n + n' : n : n' \tag{79}$$

It is easily seen that W_e is larger than W or W_m, and if the *rotor* is connected to a receiving circuit, it will give out considerable electrical power.

When driven backward in this way the machine is called a frequency changer, and generally changes current from a lower (say, fifteen cycles) to a higher (say, twenty-five cycles) frequency. It is generally driven backward by a small synchronous or induction motor connected to the low-frequency mains.

131. The Induction Generator.—If an induction motor is driven above synchronous speed, mechanical power $2\pi\, n'\, T$ must be supplied to the rotor. The electrical power developed in the rotor $W_e = 2\pi\, n'\, T - 2\pi\, n\, T$

$$\begin{aligned} W &= 2\pi\, n\, T \\ W_m &= 2\pi\, n'\, T \end{aligned}$$

But since W_m is greater than W $(W_m = W + W_e)$, part of W_m is used in supplying electrical power to the rotor and part of W_m is used in supplying electrical power to the stator, which in turn pumps it out on the line as a generator.

Therefore $$W : W_m : W_e = n : n' : (n' - n) \tag{80}$$

When so driven, the machine is called an *asynchronous generator.*

132. Summary.—

$$\text{1. Induction motor}\,\ldots\ldots\left\{\begin{array}{ll} W=2\pi\, n\, T & \text{input}\\ W_m=2\pi\, n'\, T & \text{output}\\ W_e=2\pi\, (n-n')\, T & \text{output}\\ \text{and } W:W_m:W_e=n:n':(n-n') & \end{array}\right.$$

$$\text{2. Frequency changer}\ldots\left\{\begin{array}{ll} W=2\pi\, n\, T & \text{input}\\ W_m=2\pi\, n'\, T & \text{input}\\ W_e=2\pi\, (n+n')\, T & \text{output}\\ \text{and } W:W_m:W_e=n:n':(n+n') & \end{array}\right.$$

$$\text{3. Induction generator}\,\ldots\left\{\begin{array}{ll} W=2\pi\, n\, T & \text{output}\\ W_m=2\pi\, n'\, T & \text{input}\\ W_e=2\pi\, (n'-n)\, T & \text{output}\\ \text{and } W:W_m:W_e=n:n':(n'-n) & \end{array}\right.$$

In all three cases W and W_m are always the same expression.

In all three cases W_e is always proportional to the relative speed between the stator flux (n) and the rotor (n′), and is always an output.

In order that there may be equilibrium the sum of the outputs equals the input, and vice versa.

Induction generators may be operated in parallel with ordinary synchronous machines (generators) without being synchronized, for if their speed is too low when thrown on the line, they will operate as induction motors and tend to drive the prime mover to which they are connected. (Reference: "Alternating Current Motors," A. S. McAllister, chap. viii.)

133. Rating of Induction Motors.—They are rated at a certain H.P. and speed when supplied with a certain voltage, certain frequency, and certain number of phases. Thus we speak of a certain 20 H.P. 220-volt two-phase 1120 r.p.m. motor. The synchronous speed of a six-pole motor is 1200 r.p.m.; so, allowing for slip, this is evidently a six-pole motor.

If an induction motor is overloaded until the maximum torque is exceeded (see curve $R=1$, Fig. 207), it cannot speed up, because the torque will get less and it therefore will come to rest. This is called the breaking down of the motor, and does not take place until the motor is from 50% to 150% overloaded, which rarely occurs. The curves on Fig. 212 show the breakdown point to be at 70% overload.

PROBLEMS.

178. With figures similar to Figs. 202 and 203 show the effect of reversing one of the phases of a two-phase motor, on the direction of rotation of the rotating field, and hence on the direction of rotation of the motor.

(*Note:*—Reversing any two wires on a three-phase motor will reverse the direction of the motor.)

179. Calculate the percentage slip of a four-pole sixty-cycle induction motor running at 1760 r.p.m. *Ans.:*—2.2+%.

180. A ten-pole 300 r.p.m. generator drives a 10 H.P. induction motor at full load with 4% slip, at a speed of 480 r.p.m.; find the number of poles on the motor. *Ans.:*—6 poles.

181. In problem 180, neglecting the stator losses, and windage and friction, calculate the efficiency and the torque at full load. *Ans.:*—96%; 109.4 foot-pounds.

182. An eight-pole motor on sixty cycles runs at 855 r.p.m. Calculate the slip. With the same slip what will the speed be if the poles, by a suitable switching device, are reduced to 4. *Ans.:*—5% slip, 1710 r.p.m.

183. Plot a set of starting curves similar to those in Fig. 206 for a sixty-cycle six-pole motor operating on 220-volt mains. The rotor has the same number of turns of wire as the stator and is similarly wound (1 to 1 ratio of transformation). $R_r = .1$ ohm X_r $(2\pi fL) = 1$ ohm when rotor is at rest. Calculate and plot four points for each curve (torque curve is indeterminate). Plot a second current curve for $R_r = 1$ ohm.

184. A perfect three-phase six-pole motor has its stator star connected to 240-volt sixty-cycle mains. The stator takes 5 amperes from each line and the rotor is star-connected and delivers 10 amperes through collector rings to each of three receiving circuits, each of which has a power factor of .8. Under these conditions the rotor runs at three-fourths synchronous speed. Find: (*a*) ratio of stator to rotor turns; (*b*) voltage generated in rotor; (*c*) electrical power delivered to stator; (*d*) electrical power delivered to rotor; (*e*) mechanical power delivered by rotor. *Ans.:*—(*a*) 2 to 1; (*b*) 30 volts; (*c*) 16,600 watts; (*d*) 4150 watts; (*e*) 12,450 watts.

185. The motor in problem 184 is driven backward at a speed of 1467 r.p.m. The stator is still Y-connected, but the rotor is connected through collector rings to three similar receiving circuits; the receiving circuits are Y-connected with each other and each has a resistance of .6 ohm and a reactance of .8 ohm. Find (*a*) the frequency and the current flowing in each rotor circuit; (*b*) the electrical power delivered by the stator; (*c*) the electrical power delivered by the rotor; (*d*) the mechanical power delivered to the rotor. *Ans.:*—(*a*) 133 cycles 51.3 amperes; (*b*) 2880 watts; (*c*) 6400 watts; (*d*) 3520 watts.

186. The motor in problem 185 is connected to the same receiving circuit, but is driven forward at a speed of 2667 r.p.m. Find: (*a*) the frequency and current in each rotor circuit; (*b*) the electrical power delivered by the stator to the line; (*c*) the electrical power delivered by the rotor; (*d*) the mechanical power delivered to the rotor. *Ans.:*—(*a*) 133 cycles 51.3 amperes; (*b*) 5236 watts; (*c*) 6400 watts; (*d*) 11.636 watts.

187. Make a diagrammatic sketch similar to Fig. 210 for starting a two-phase induction motor.

CHAPTER XII.

SINGLE-PHASE MOTORS.

134. Construction of the Single-phase Induction Motor.—The stator is entirely similar to the stator of a two-phase induction motor, and is of course laminated. Phase A is called the operating winding, and phase B the starting winding, which is automatically cut out as the motor speeds up. The rotor on small motors is generally of the squirrel-cage type, and on larger motors there is often a squirrel-cage high-resistance winding and a three-phase low-resistance winding, both placed in the same slots. The motor starts with only the high-resistance winding operative, which limits the starting current. As the motor speeds up, the low-resistance winding is automatically short-circuited and both rotor windings act together. On some squirrel-cage rotors the slots are skewed to prevent humming. Many small fan motors have individual field poles similar to D.C. motors.

135. Theory of Operation of the Single-phase Induction Motor.—For purposes of discussion we will assume a two-pole single-phase induction motor, and at right angles to these two poles we will draw two other poles, A B, with no windings on them (Fig. 213). With the direction of flow of current in the stator as shown, the stator, or primary field (ϕ), as we shall call it, will be *down*. Let us further assume that the rotor is revolving with a speed n′

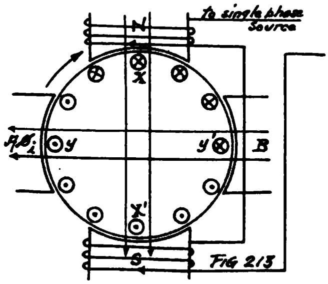


FIG 213

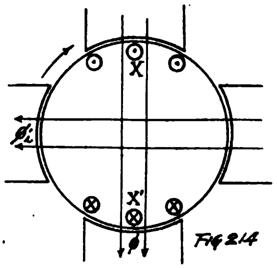


FIG 214

in the direction indicated by the arrow; evidently the conductors on the rotor in the region XX′ will have an induced e.m.f. due to the rotation of the rotor (X_r), which will be "in" at X and "out" at X′, and have the same frequency as ϕ.

It must be clearly kept in mind that Fig. 213 represents the mechanical positions of the fluxes and currents and bears no relation whatever in position to Fig. 215, which shows the phase relations in the motor.

As in the case of the transformer (see Fig. 174), we have drawn the primary flux ϕ along the horizontal (Fig. 215). With constant speed X_r varies in magnitude with the magnitude of ϕ; in other words, X_r is in phase with ϕ (Fig. 215).

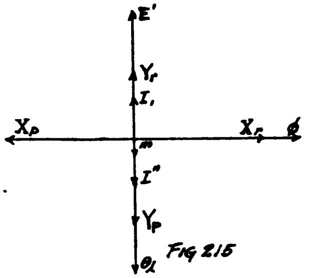


Fig 215

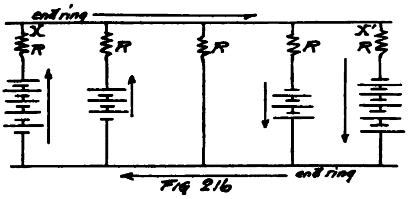


Fig 216

The conditions of parallel and series circuits which exist in squirrel-cage rotors are shown by the analogy in Fig. 216. In the region X which corresponds to X (Fig. 213) there is an e.m.f. E_r induced in a given direction, and in the region X′ there is an e.m.f. E_r generated in the opposite direction; these e.m.f.'s taper off to zero value between X and X′, as shown by the circuit with no battery; it is seen that most of the current flows in the regions X X′. It is also seen that the current in the end rings of the squirrel-cage is reversed once with respect to the field poles for every reversal of ϕ, or once every cycle.

In the region Y Y′ there will be induced in the rotor an e.m.f. Y_p, which is caused by the pulsations of ϕ, and of course this e.m.f. is 90° behind ϕ (Fig. 215). (The e.m.f. induced in a circuit due to pulsation of flux ($2\pi fLI$) is 90° behind the flux as E″ in Fig. 174.)

The "in"-flowing currents in X and the "out"-flowing currents in X′ produce an induced or secondary field, ϕ_i, in the region A B from right to left, which has the same frequency as ϕ. Since this latter field, ϕ_i, must have a rate of change approximately equal to the e.m.f. (X_r) generated in the region X X′ by the cutting of the primary field, it follows that ϕ_i must be 90° behind X_r (Fig. 215), because the flux in any circuit is always 90° behind the e.m.f. producing it ($2\pi fLI$), and 90° ahead of the e.m.f. induced by it ($2\pi fLI$) (see Fig. 56).

In the region X X′ there will be induced in the rotor an e.m.f. X_p which is caused by the pulsations of ϕ_i, and of course this e.m.f. is 90° behind ϕ_i (Fig. 215), and is opposite and nearly equal to X_r.

In exactly the same way ϕ_i will produce an e.m.f. Y_r in the region Y Y′ due to rotation Y_r (Fig. 215), which is opposite and nearly equal to Y_p. If the motor is running at synchronous speed (n), then $Y_r = Y_p$, and the only current flowing in the rotor is the current necessary to produce ϕ_i, and therefore in phase with ϕ_i, and called the magnetizing current, m (Fig. 215; also see Fig. 174). If the rotor runs below synchronous speed n, say at speed n′, then Y_r is less than Y_p, and a current flows in the rotor in phase with Y_p and called the load current I″. The magnetizing action of this load current I″ is balanced by an opposing load current I′, in the stator winding.

It is evident that ϕ is independent of the rotor speed n′, since it must be sufficient to produce enough b.e.m.f. in the stator to balance the impressed e.m.f. on the stator.

ϕ_i, however, is proportional to n′, and only equals ϕ when n′ equals n. This can be seen from the fact that X_r, due to the cutting of ϕ with a speed n′, must equal X_p, due to the pulsating of ϕ with a speed n.

The two fluxes ϕ and ϕ_i, at right angles to each other and 90° apart, produce a rotating resultant field, just as in the polyphase motor; and, for the same reason, the motor revolves in the direction of that rotating flux.

If in the discussion above the direction of motion had been counter-clockwise, this would have reversed the direction of ϕ_i, and hence the direction of rotation of the rotating field and of the motor. If in the above discussion the motor had remained stationary, then none of the currents, voltages, or fluxes of Fig. 215 would have existed except ϕ and Y_p, and therefore there would be no torque.

It should be noticed that the existence of ϕ and ϕ_i at right angles in the same material is entirely imaginary. The resultant field is real, however, and the effect of ϕ and ϕ_1 is real.

We can see that a single-phase induction motor is not self-starting, but if once started will continue to run in either direction that it is started; it therefore follows that a polyphase motor will continue to run on one phase, but of course will take more current than when all the phases are in use.

Since the single-phase motor is not self-starting, its speed torque curve (see Fig. 207) passes through zero at zero speed, and when starting devices are used the starting torque is much smaller than in a polyphase machine. (See S. A. McAllister, "Alternating Current Motors," pp. 138–148.)

136. Methods of Starting Single-phase Induction Motors.—There are three methods commonly used:

First: Phase-splitting devices, generally used with the larger sizes.

Second: Shading coils, used with fan and other small motors.

Third: As a repulsion motor where a large starting torque is required.

1. *Phase Splitters.*—With this method of starting the motor has two windings placed on its stator, like a two-phase induction motor.

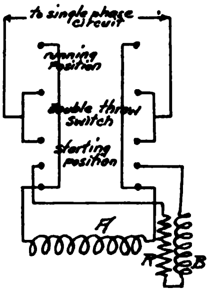


Fig 217

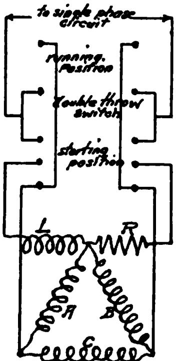


Fig 218

A suitable resistance or condenser is placed in series with phase B. Evidently the current in phase A will lag behind the current in phase B (Fig. 217), and thus an imperfect (elliptical) rotating field will be produced sufficient to start the motor. At about half speed the switch is thrown to the running position and phase A only is operating. Fig. 217 shows likewise a method for using a two-phase motor on a single-phase circuit.

The windings A and B (Fig. 217) are frequently so proportioned that the phase displacement is caused by the windings themselves. Phase A is called the operating winding and phase B the starting winding, in which case no resistance R (Fig. 217) is needed.

Stator windings are often arranged in two groups. When these groups are connected in series, the motor is suitable for, say, 220 volts; and when connected in parallel, the motor is suitable for 110 volts. This is also true of small polyphase motors.

Because of the small starting torque, an automatic clutch is provided which permits the motor to take up its load as it approaches normal speed. The direction of starting may be reversed, as in any polyphase induction motor.

Fig. 218 shows a method of using a three-phase motor on a single-phase circuit. It will be noticed that in the running position phase C only is operating.

2. *The Shading Coil.*—This method is extensively used in connection with small fan motors, but is not economically applicable to large motors. The motor has field poles similar to those in multipolar D.C. motors, and across the middle of each pole face is cut a slot parallel to the shaft. In this slot and around one half the pole face is placed a rectangular copper strips (Fig. 219, a) of extremely low resistance.

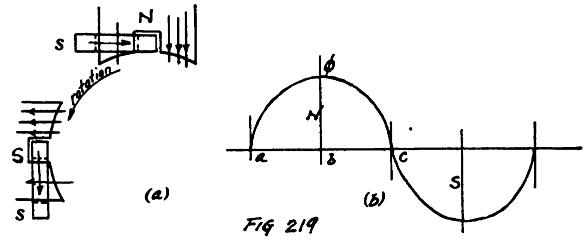


Fig 219

In Fig. 219 are drawn two poles of a four-pole motor with the shading coils s s as shown. Let us consider the instant a (Fig. 219, b) when the upper pole is just becoming a N pole. Since ϕ is changing, there will be an e.m.f., and hence a current induced in s which will be proportional to the rate of change of ϕ, and in such a direction as to oppose any change in ϕ, which means that most of the increasing flux will be in the unshaded portion of N, so that the flux distribution over N will be as shown.

At the instant b (Fig. 219, b), ϕ is not changing in value, therefore there is no e.m.f. and no current induced in s, and no back flux produced by s, and since s is inactive the flux distribution over N will be uniform.

At the instant c (Fig. 219, b), N has just lost its magnetism and ϕ is dying away, and since ϕ is again changing, but in the opposite direction to what it was at a (Fig. 219, b), there will be induced in s an e.m.f., and therefore a current opposite in direction to what they were at a (Fig. 219, b), and therefore in such a direction as to prevent the dying away of ϕ, which will make the unshaded portion of N have no flux and the shaded portion a N pole.

At instant a the N pole was on the unshaded portion, as shown in Fig. 219, a; at instant b, the N pole was fairly well distributed over the entire pole face; at instant c, the N pole was on the shaded portion. Thus we see that the flux is varying in value and constantly shifting counter-clockwise. This irregular rotating field is sufficient to start the motor, which, of course, revolves in the same direction. If the shading coils were placed on the other half of the poles, it can similarly be shown that the direction of motion would be reversed.

3. *Starting as a Repulsion Motor.*—(See repulsion motor.)

137. The Repulsion Motor.—The repulsion motor has its fields excited by single phase A.C., and its armature is provided with an ordinary direct-current winding.

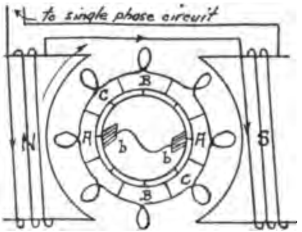


Fig 220

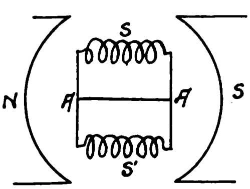


Fig 221

Fig. 220 represents a two-pole repulsion motor; it is shown with an eight-coil ring-wound armature with a commutator and two brushes, b b, which are short-circuited on each other. Consider the brushes under the coils A A (Fig. 220); this condition is shown diagrammatically in Fig. 221. The two sides S S′ of the armature are now acting as the short-circuited secondaries of a transformer; and since their axes are parallel to the axes of the primary, no torque is exerted but the e.m.f., and therefore the current induced in the armature is a maximum.

Consider the brushes under the coils B B (Fig. 220); this condition is shown diagrammatically in Fig. 222. Here the total e.m.f. induced in the secondaries S S′ is zero, since their axes are at right angles to the axes of the primary, and therefore the current in the armature is zero, and so is the torque zero.

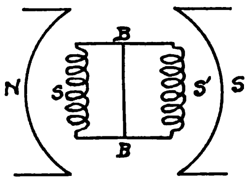


Fig 222

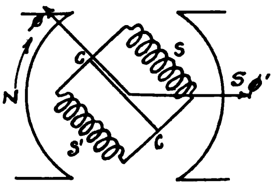


Fig 223

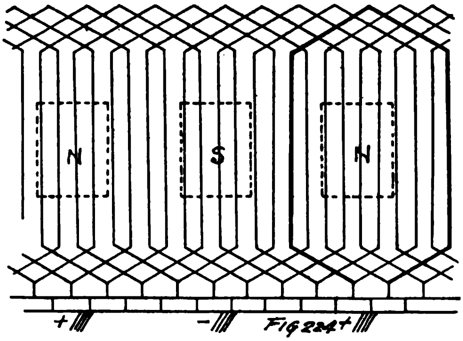
Again consider the brushes under the coils C C (Fig. 220). This condition is shown diagrammatically in Fig. 223. The two sides, S S′, of the armature are again acting as the short-circuited secondaries of a transformer. Now their axes are 45° from the axes of the primary, and therefore some primary flux will thread the secondaries and some e.m.f., and therefore some current will flow in them. At the instant shown the primary flux is from left to right (Fig. 223). Ordinarily in a transformer ϕ'' is 180° from ϕ' (see Fig. 174), but since primary and secondary are not parallel, but 45° apart, ϕ'' and ϕ' will not be 180° apart, but only 135° (see Fig. 223). Since two fields always try to coincide, and ϕ' is fixed, ϕ'' in its effort to coincide with ϕ' will exert torque in a clockwise direction on the armature.

The actual motor has many more than eight coils on its armature, and to keep the current in the armature from becoming excessive the brushes are placed very near the position B B (Fig. 220), although any position of the brushes but A A and B B will make the motor self-starting. To reverse the motor it is only necessary to shift the brushes from the one side of the B B position to the other.

As the machine quickly speeds up a mechanical device raises the brushes from the commutator and a ring is pressed against the commutator, thus short-circuiting the armature, and then the machine runs as a single-phase induction motor with a squirrel-cage rotor.

This entire discussion may be carried through for the short-circuited coils A and B and C (Fig. 220), with exactly the same conclusion, and since the torque is exerted only when coil C is short-circuited, and since the direction of motion is clockwise, coil C is said to be repelled by the field N; hence the name repulsion motor. (This motor is made by the Wagner Co., and for construction of same, see I. C. S. pamphlet, "Alternating Current Apparatus," pp. 59–64.)

138. The Single-phase Series Motor.—(*a*) *The D.C. Shunt Motor on an A.C. Circuit.*—When so used, the shunt motor would probably run, because the current in the armature is reversed every time the current in the field is reversed (if the currents in the armature and field of a D.C. motor are both reversed, the motor will continue to run in the same direction), but it will not work satisfactorily because the field current will lag too much behind the armature current, owing to its high inductance (so many turns of wire), and thus the motor will have but little torque.



(*b*) *The Series Motor on an A.C. Circuit.*—If a series motor is used on an A.C. circuit, it will produce a continuous torque, because its armature current is reversed every time its field current is reversed, and it has the added advantage that these two currents must be in phase with each other, since armature and field are connected in series. When the current is passing through zero, the torque will be zero, so it can be seen that the torque is not uniform, but varies through a set of values from 0 to a maximum.

139. Sparking in Single-phase Series Motors.—The greatest difficulty encountered in the development of the series motor for use on A.C. circuits has been the excessive sparking at the brushes.

In Fig. 224 the brushes have been placed exactly as they would be if run on D.C. The brush on the right short-circuits the coil shown heavy; since this coil has its sides between pole faces, it is not cutting any flux, and therefore there is no voltage generated in it, due to the rotation of the armature. This e.m.f. is called *rotational e.m.f.*, and is the only e.m.f. in D.C. machines.

The heavy coil short-circuited by the brush entirely surrounds the N pole, and therefore this coil acts like a short-circuited secondary of a transformer, and is in a position to have induced in it a maximum e.m.f. due to the pulsation of the field flux. This e.m.f. is called the *pulsational e.m.f.*, and is the only e.m.f. present in transformers.

This pulsational e.m.f. causes a large short-circuited current to flow in the coil shown heavy, thus producing bad sparking at the brushes.

The pulsational e.m.f. is a maximum when the rotational e.m.f. is zero.

140. Methods of Preventing Sparking.—Many methods for preventing sparking have been devised, but the only one presented will be that used by the Westinghouse Company.

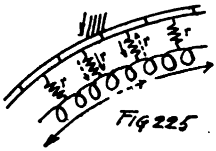


Fig 225

Resistance leads (r, r, r, r, Fig. 225) are inserted between the armature windings and the commutator. Evidently the resistance to the flow of the main current is ½ r, as shown by the full-line arrows in the figure; the resistance to the flow of the short-circuited current is 2 r, as shown by the dotted arrows. These resistances are so proportioned that the short-circuited current is equal to the normal current. These leads only carry the currents during the instant a brush is sliding over a commutator bar, and then successive leads take it up so they are designed to carry the current temporarily. If the armature should stick at starting, two coils might carry the current long enough to be injured or burned out. To prevent such an occurrence the leads are made up of large current-carrying capacity, and are generally heavy strips of German silver carefully insulated and placed in the bottom of the armature slots; this, of course, necessitates an increase in the size of the slots, and hence of the armature.

This method also has the disadvantage of reducing the efficiency (I^2R losses are increased), and to keep these losses as small as possible the brushes should be as narrow as practicable, thus decreasing the duration of the short circuit. (See S. A. McAllister, "Alternating Current Motors," chap. xvi.)

141. The Clock Diagram of a Single-phase Series Motor.—Since a series motor is a series circuit, current I is common to the circuit, and is therefore drawn as the reference axis (Fig. 227).

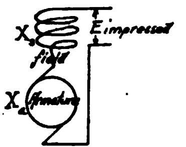


FIG 226

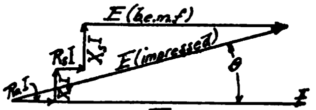


Fig 227

The voltages to overcome armature resistance (R_aI) and field resistance (R_sI) are in phase with I, and the voltages to overcome armature reactance (X_aI) and field reactance (X_sI) are 90° ahead of I.

Let c be the number of conductors on the armature
ϕ be the alternating field flux
and n be the speed of rotation
Now E (b.e.m.f.) $= \phi$ cn (fundamental equation of a D.C. dynamo or motor)
and since c and n are constant
E (b.e.m.f.) is in phase with ϕ
but ϕ is in phase with I
therefore since E is a back e.m.f., the position of the impressed e.m.f. to overcome it is in phase with I and represented by the line E (b.e.m.f.) in Fig. 227. E (impressed) is the vector sum of the five vectors shown, and cos θ is the power factor of the motor.

142. Methods of Improving the Power Factor.—1. By increasing I_aR and I_sR; this, however, lowers the efficiency.

2. By increasing E (b.e.m.f.); this can be done by using a large number of conductors on the armature (large value of c in equation E (b.e.m.f.) $= \phi$ c.n.).

3. By decreasing X_sI $(= 2\pi f L_s I)$; this can be accomplished by using a low-frequency f (not over 25); this will also reduce the

sparking. And L_s may be reduced by having a small air-gap and a low flux density, so that the required flux may be obtained with a minimum number of ampere turns on the field. This means a larger field core than on the corresponding D.C. motor.

4. By decreasing X_aI ($=2\pi fL_aI$); this is produced by using a low-frequency, and can, in fact, be completely annulled by the use of a compensating winding. (See art. 143.)

143. Compensated Series Motor or the Neutralizing of the Armature Inductance (L_a).—If three turns of wire carrying one ampere are wound around a closed magnetic circuit, lines of force (inductance L) exist in the iron (Fig. 228, a); but if three more turns of wire carrying one ampere are wound around the same circuit with current flowing in the opposite direction (Fig. 228, b), the two coils annul each other, and no lines of force exist in the iron. (Inductance L = O.) This is the principle involved in compensating for armature inductance or for reducing X_aI to zero.

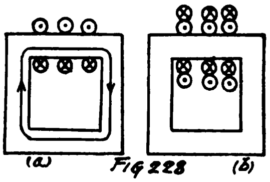


Fig 228

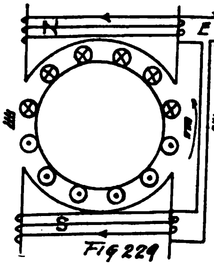


Fig 229

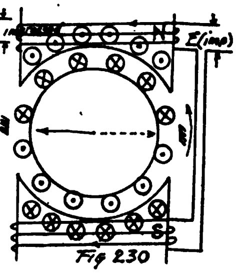


Fig 230

Consider the two-pole motor (Fig. 229). At a given instant the N pole is above, the S pole is below, and current is flowing "in" at the top and "out" at the bottom of the armature. These armature currents will cause a flux to be produced in the armature from right to left. One half-cycle later the upper pole becomes a S pole; the armature current is flowing "out" at the top and the armature flux is from left to right, as shown by the dotted arrow (Fig. 230). Evidently this armature flux could be neutralized if the same number of turns as are on the armature are embedded in the pole faces, as shown in Fig. 230, and provided these turns are connected in series with the motor, with the current in them 180° from the armature current. Such an arrangement is called *forced compensation*, and is shown diagrammatically in Fig. 231.

The compensated winding has its axis 90° from the axis of the field coils. If the compensated winding were short-circuited on itself, it would act as the secondary of a transformer with the armature as the primary, and, having its current I″ 180° from the armature current I′, would produce the same result. This arrangement is called *transformer compensation*, and is shown diagrammatically in Fig. 232. In this case the number of turns on the compensating winding does not necessarily equal the number of turns on the armature.

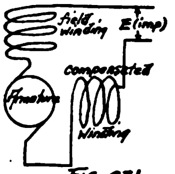


Fig 231

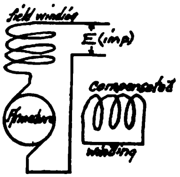


Fig 232

Fig. 233 represents a four-pole compensated motor; the compensating winding is connected as shown in Fig. 234. If the ends xx′ are connected together, it is transformer or inductive compensation; if x and x′ are connected in series with the motor, as in Fig. 231, it is forced or conductive compensation.

144. The Series Repulsion Motor.—This motor embodies the best features of the repulsion motor and of the compensated series motor. Fig. 235 shows diagrammatically the motor as arranged

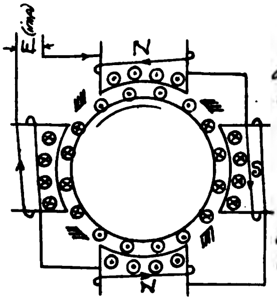


Fig 233

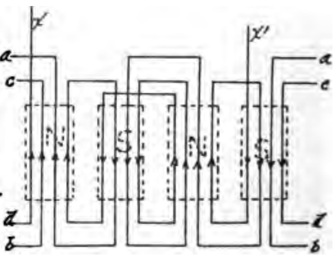


Fig 234

for starting. The armature is short-circuited, and it starts as a repulsion motor. The compensating winding has twice as many turns as are on the armature; this allows the short-circuited armature to have twice the starting current that the field has.

This, of course, gives twice the torque of the compensated series motor. When in the running position the short circuit on the armature is removed, and it then has the same characteristics as the compensated series motor. It is claimed that this motor possesses less sparking than the latter.

145. Uses of the Series-compensated and Series-repulsion Motors.—They are used almost exclusively for railway work.

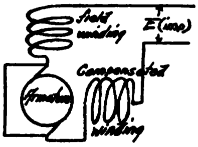


Fig 235

The old system is to generate at, say, 11,000 volts, and then have substations located every few miles, and in these have step-down transformers and rotary converters and transform to D.C. at 550 or 600 volts, which is delivered to the trolley.

With the use of the above series motors the trolley line takes the 11,000 volts generated and serves as the transmission line, and with a transformer on the car, the voltage is reduced to about 220, the voltage of the motor. This does away with the need of substations.

Since these motors are started under load, their voltage must be reduced at starting; this is not done by a complicated controller and resistance grids with their heavy I^2R losses, but by having a number of taps on the secondary of the step-down transformer. The motor does not have as high efficiency as the D.C. series motor or as good a torque at start, and because of the large amount of copper on its armature and the large amount of iron in its field (due to low flux density) a 200 H.P. alternating current motor is much larger than a 200 H.P. direct-current motor.

Some systems are now arranged so as to operate on high voltage A.C. on private rights of way between cities, and then use the D.C. system when passing through the cities.

REFERENCES.

S. A. McAllister: "Alternating Current Motors," chapter xii to xv inc.
D. F. Punga: "Single-phase Commutator Motors."
D. C. and J. P. Jackson: Alternating Currents, art. 178, "Electro-magnetic Repulsion Motor."

CHAPTER XIII.

CONVERTERS.

The two methods, most commonly used, of converting A.C. into D.C. are, first, the rotary or synchronous converter; second, the motor generator. There might also be mentioned the mercury arc rectifier and the rectifying commutator which has been described. (See Fig. 148.)

146. The Rotary Converter.—This machine has a D.C. field and an ordinary D.C. drum armature winding which is connected at one end of the shaft to a D.C. commutator with brushes, and at the other end of the shaft to A.C. slip rings with brushes. When the A.C. brushes are connected to an A.C. source of supply, the machine will operate as a synchronous motor, and when so running, direct current may be taken from the commutator end. The input to the A.C. side is greater than the D.C. output by the copper losses, windage, and brush and bearing friction losses.

The taps to each A.C. ring must occur every pair of poles (every 360°). A converter may have any number of rings greater than one. A converter with n-rings is called an n-ring converter. A single-phase converter has two rings, a two-phase converter has four rings, a three-phase converter three rings, and a six-phase converter six rings.

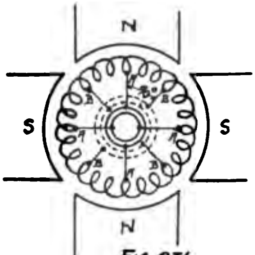


Fig 236

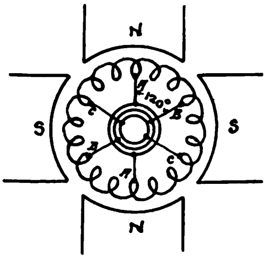


Fig 237

In Fig. 236 the full lines represent the two connections to each ring for a four-pole single-phase rotary. The addition of the dotted rings, with the connections thereto, makes a two-phase rotary. It will be noticed that phase B is 90 electrical degrees from phase A.

In Fig. 237 are shown the two connections to each of the three rings of a four-pole three-phase rotary.

147. The Voltage and Current Relations in a Rotary Converter Armature.—(*a*) The single-phase converter.

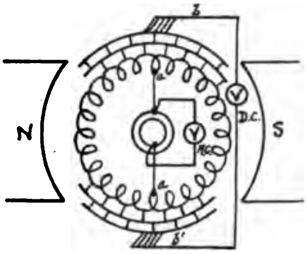


Fig 238

Voltage relations: Let Fig. 238 represent a two-pole single-phase converter. Suppose that the D.C. voltmeter is reading 100 volts. The A.C. voltage is going through a set of positive and negative values, but at the instant the ring connections a a′ are in the vertical position shown, the A.C. voltage is a maximum; but in the position shown, a and b are connected together, and a′ and b′ are connected together, therefore the two voltmeters are connected together and read the same. Hence we have the relation:—

$$E_{(\mathrm{max})} = E_{(\mathrm{D.C.})}$$

assuming a sine wave

$$E_{(\mathrm{r.m.s.})} = \frac{E_{(\mathrm{max})}}{\sqrt{2}} = \frac{E_{(\mathrm{D.C.})}}{\sqrt{2}}$$

therefore

$$E_{(\mathrm{1\text{-}ph})} = E_{(\mathrm{A.C.})} = \frac{E_{(\mathrm{D.C.})}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ volts} \tag{81}$$

Current relations: Neglecting all losses

we have

$$E_{(\mathrm{A.C.})}\, I_{(\mathrm{A.C.})} = E_{(\mathrm{D.C.})}\, I_{(\mathrm{D.C.})}$$

or

$$\frac{E_{(\mathrm{D.C.})}}{\sqrt{2}}\, I_{(\mathrm{A.C.})} = E_{(\mathrm{D.C.})}\, I_{(\mathrm{D.C.})}$$

or

$$I_{(\mathrm{1\text{-}ph})} = I_{(\mathrm{A.C.})} = \sqrt{2}\, I_{(\mathrm{D.C.})} \text{ if } I_{(\mathrm{D.C.})} = 100 \text{ amp's, } I_{(\mathrm{1\text{-}ph})} = 141.4 \text{ amp's} \tag{82}$$

(*b*) The two-phase converter:

Voltage relations: Since the two-phase rotary has the taps for each phase 180° apart, just as in a single-phase rotary, it is clear that the voltage relations remain the same.

$$E_{(2\text{-ph})} = E_{(A.C.)} = \frac{E_{(D.C.)}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ volts}$$

Current relations: Neglecting all losses

we have $\quad 2\,E_{(A.C.)}\,I_{(A.C.)} = E_{(D.C.)}\,I_{(D.C.)}$

or $\quad 2\,\dfrac{E_{(D.C.)}}{\sqrt{2}}\,I_{(A.C.)} = E_{(D.C.)}\,I_{(D.C.)}$

or $$I_{(2\text{-ph})} = I_{(A.C.)} = \frac{I_{(D.C.)}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ amperes} \qquad (83)$$

(*c*) The three-phase converter:

Voltage relations: Taps a, b, and c (Fig. 239) show the connections for a three-phase converter, and taps a and d for a single-phase converter. Evidently $E_{3\text{-ph}}$ is smaller than $E_{1\text{-ph}}$ by the ratio of ac to ad.

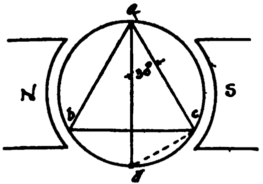


Fig 239

By geometry $\quad ac = ad \cos 30°$

$\quad ac = ad\,\dfrac{\sqrt{3}}{2}$

or $$E_{(3\text{-ph})} = \frac{\sqrt{3}}{2}\,E_{(1\text{-ph})} = \frac{\sqrt{3}}{2}\,\frac{E_{(D.C.)}}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}}\,E_{(D.C.)} = 61.2 \text{ volts} \qquad (84)$$

Current relations: Neglecting all losses

we have $\quad 3\,E_{(A.C.)}\,I_{(A.C.)} = E_{(D.C.)}\,I_{(D.C.)}$

or $\quad 3\left(\dfrac{\sqrt{3}}{2\sqrt{2}}\,E_{(D.C.)}\right) I_{(A.C.)} = E_{(D.C.)}\,I_{(D.C.)}$

or $$I_{(3\text{-ph})} = \frac{2\sqrt{2}}{3\sqrt{3}}\,I_{(D.C.)} = \frac{2\sqrt{2}}{3\sqrt{3}} \times 100 = 54.4 \text{ amperes} \qquad (85)$$

Summary:—

Single-phase	*Two-phase*	*Three-phase*
$E = \dfrac{E_{(D.C.)}}{\sqrt{2}}$	$E = \dfrac{E_{(D.C.)}}{\sqrt{2}}$	$E = \dfrac{\sqrt{3}}{2\sqrt{2}}\,E_{(D.C.)}$
$I = \sqrt{2}\,I_{(D.C.)}$	$I = \dfrac{I_{(D.C.)}}{\sqrt{2}}$	$I = \dfrac{2\sqrt{2}}{3\sqrt{3}}\,I_{(D.C.)}$

From these relations the A.C. line currents can easily be obtained.

148. Heating of Rotary Converters.—Since the A.C. side of a converter is a motor and the D.C. side a generator, and since both have the same field poles, it follows that the A.C. and D.C. currents flowing in the armature are 180° apart; since the heating effect depends on the net current flowing, a machine when used as a converter will have less heating in its armature than if run as a D.C. generator.

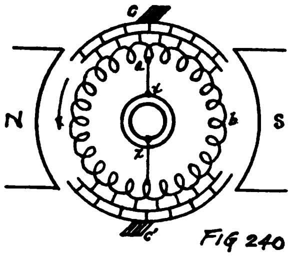


Fig 240

The actual currents that flow in the armature are the difference between the alternating current supplied and the direct current given out.

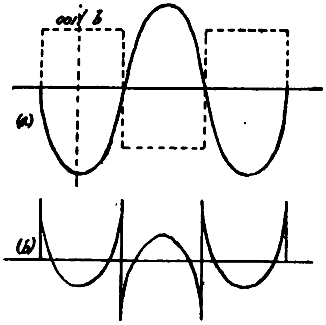


Fig 241

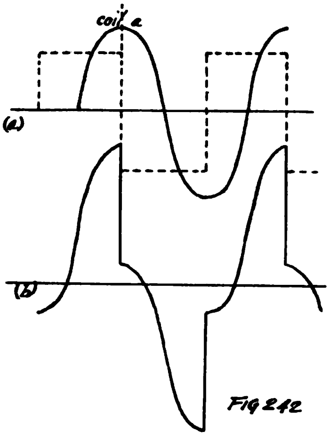


Fig 242

Consider the single-phase rotary (Fig. 240) at the instant the A.C. ring connections are passing under the brushes c c'. At the instant a coil (a) passes under a brush its direct current is reversed, but while the coil is passing through the half revolution from c to c′ the direct current is not reversed.

Let us consider the currents flowing in the coil b (Fig. 240) situated midway between the tap points x x. The direct current in this coil b may be represented by the rectangular dotted wave (Fig. 241, a). The alternating current in the entire half of the armature from c around to c′, and of course including coil b, is a maximum at the instant shown (when the points x x are directly beneath the brushes), and will become zero at the instant coil b passes under brush c. In other words, the direct current in coil b will be reversed at the same instant as the alternating current, and therefore the A.C. wave of current in coil b (Fig. 240) may be represented by the sine wave (Fig. 241, a) which passes through zero at the same instant as the direct-current wave. Fig. 241, b, shows the resultant-current wave which has a r.m.s. value less than the direct-current wave, and therefore coil b will not get as hot when the machine is operating as a converter as when running as a D.C. dynamo or as a synchronous motor, since the heating varies as the square of the r.m.s. value.

Again, let us consider the coil a (Fig. 240), which is the coil directly at one of the A.C. ring connections x, and at the instant shown in Fig. 240 is passing under brush c. The direct current in coil a is, as before, a rectangular wave, but is just being reversed. (See Fig. 242, a.) The alternating current in coil a, like the alternating current in coil b and in the entire half of the armature, is a maximum, because the ring connections are vertical, and the A.C. sine wave is therefore 90° from the D.C. rectangular wave, as shown in Fig. 242, a. Fig. 242, b, shows the resultant-current wave, and it is seen that Fig. 242, b, has a larger r.m.s. value than the rectangular wave. Therefore the coil a will run hotter than if the machine be run as a D.C. generator. For this reason a single-phase converter will not give as large a K.W. output as if it were run as a D.C generator, and the hottest coils are those next to the ring connections (coil a).

As the number of ring connections, and hence the number of phases, is increased, the more uniform does the heating become, and the less it becomes as compared to a D.C. generator. It is for this reason that the greater the number of ring connections, the greater will be the output of a given machine.

TABLE OF CAPACITIES OF A CONVERTER.

Used as a D.C generator	10.0 K.W.	
Used as a Single-phase converter	8.5 K.W.	
Used as a Three-phase converter	13.4 K.W.	
Used as a Two-phase (4 taps) converter	16.4 K.W.	(often called four-phase)
Used as a Six-phase converter	19.6 K.W.	

Single-phase converters are never used, and one of the great advantages of a polyphase machine is its increased output. The output of a converter used as such is much greater than when used as a D.C. generator; this explains the fact that a converter has a larger commutator than a D.C. generator of the same output.

149. Armature Reactions in Converters.—When a rotary converter is running steadily (no hunting) at any load, there is only enough torque exerted to overcome mechanical and electrical friction. Therefore the field flux is not much distorted and the D.C. brushes are placed so as to commutate the current in coils midway between successive poles. The direct-current armature reaction is mostly cross-magnetizing action, which tends to distort the field.

When the A.C. current and voltage are in phase with each other, the A.C. armature reactions are mostly cross-magnetizing action, which tends to distort the field in the opposite direction. (See art. 93.) The result of the one reaction neutralizing the other is a field fairly free from distortion and with a uniform distribution of flux at all loads.

Equalizer rings are sometimes used to equalize the current in the different armature circuits, thus securing a uniform magnetic density under the poles pieces.

For these reasons converters have less sparking than D.C. machines, and with ordinary care no change of brush lead is necessary under varying load.

150. The Six-phase Converter.—The greatest advantage of the six-phase converter is in the increased output, due to the decrease in armature copper loss. (See table, art. 147.) Because of the greater complication of transformer and collector connections, this advantage is more than offset, and six-phase rotaries are rarely built in units smaller than 500 kilo-volt-amperes. (Reference: "Six-phase Converters," A. S. McAllister, "Alternating Current Motors," pp. 174–180; also G. E. bulletin No. 4868, p. 20.)

Fig. 243 represents a three-phase system supplying a six-phase converter. The primaries of the step-down transformers are generally Y-connected. On the secondaries A and A′ are 180° apart, and so they are on the converter. On the secondaries points A, B, and C are 120° apart, and so they are on the converter.

151. Hunting.—Since the A.C. side of a converter is essentially a synchronous motor, it is subject to hunting, like any synchronous machine. The angular velocity of the converter being alternately above and below that corresponding to synchronous speed, may at times become cumulative and quickly reach disastrous proportions. This hunting may be damped out by the use of copper bars set into the pole faces. (See methods of preventing hunting, art. 90.)

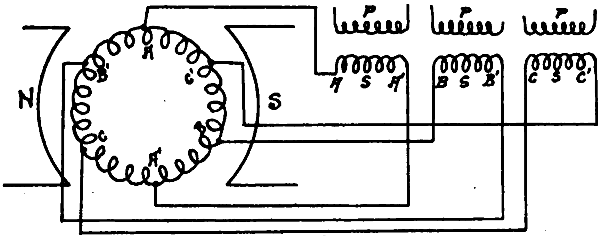


Fig 243

152. Synchronizing.—The methods employed are exactly the same as those for synchronizing two generators, or a generator and a synchronous motor, and have been explained in art. 95.

***153. Starting Rotary Converters.**—Three methods of starting rotary converters are used, viz.:—

(*a*) From the alternating-current side, as an induction motor.

(*b*) From the direct-current side, as a shunt motor.

(*c*) By means of a direct-connected starting motor, usually of the squirrel-cage induction type.

Methods (*a*) and (*c*) also apply to synchronous motors.

(*a*) *Starting from the Alternating-current Side.*—This is the method most commonly used. It consists in applying a suitable alternating electromotive force at the collector rings; the resulting torque will bring the rotary up to synchronous speed in from thirty to sixty seconds. The proper value of impressed voltage for starting is usually obtained from taps on the secondaries of the main transformers. This method renders synchronizing unnecessary.

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*Art.'s 153, 154, 155, and 156, with a few changes, are taken from Westinghouse bulletins 7038 and 1028.

The rotary is provided with a double-throw field switch, so that during starting the field may be opened in several places, thus reducing the strain upon the insulation that might otherwise result from the high potential induced in the field coils. When the alternating-current switches are closed on a self-starting rotary, the rotary is virtually a static transformer, the armature winding acting as a primary and the field coils as the secondary element. This transformer action becomes less and less as the speed increases, and entirely disappears at synchronism; but at the instant of starting the potential induced in the field coils may easily reach a dangerous value unless the field circuit is opened as described.

By throwing the field break-up switch from one position to the other the operator can reverse the field in case the rotary comes up to speed with the wrong polarity. Self-starting rotaries are also generally equipped with a switch for disconnecting the series shunt from the series field, during starting, so as to prevent the circulation of an induced current, which would otherwise produce a braking effect. Rotaries are provided with a cage damper winding, the primary function of which is to prevent "hunting," but which also acts as a starting winding. The winding consists of copper bars embedded in partially closed slots in the faces of the field poles, with the ends of these bars short-circuited in the usual manner by a copper strap. (See Fig. 165.)

(*b*) *Starting from the Direct-current Side.*—For this method of starting direct current is obtained either from the regular service bus bars or from a separate generator or circuit, and the converter is started as a shunt motor. If the converter is compound wound, the series field coils must be short-circuited or the starting circuit must be connected inside of the series coils in order that changes in armature current will not exaggerate the changes in speed by changing the field strength.

The operation of starting is similar to that of any shunt motor. The converter is started with strong field and resistance in the armature circuit provided by a starting rheostat. The converter is brought into synchronism with the alternating-current circuit by adjusting the speed by the field rheostat, and when synchronism is reached the converter is thrown in parallel with the alternating-current circuit by throwing in the A.C. switches on the high-tension or low-tension side of the transformers, as the case may be.

Certain difficulties have shown up in practice with this method of starting, but these difficulties have been, in general, due to the characteristics of the converter itself and to unsuitable starting devices. The principal difficulty is that when the converter is brought up to practically synchronous speed, any slight variations in speed change the frequency at the alternating-current end of the converter. On account of this, the synchronizing period may become so short that an ordinary operator cannot throw the switches quickly enough to insure proper paralleling. Another difficulty is that if the converter is slightly out of its true synchronous position when the alternating-current switches are closed, there will be an excessive interchange of current between the alternating- and direct-current sides of the converter.

Both of these difficulties are due to the low resistance of the armature circuits of the rotary converter. They may be overcome by inserting in the starting circuit a permanent resistance that will limit to a relatively insignificant amount the current due to ordinary changes in armature voltage. An example will serve to show the truth of this. Assume, first, a 1000 K.W. rotary converter wound for 600 volts on the direct-current side and with a normal current rating of 1666 amperes. The resistance of the armature winding will be such that about 10 K.W. will be lost in the winding with full-load current as a direct-current machine. In other words, the resistance of the armature windings is such that 6 volts is required to drive full-load current through it. Assuming that this converter is running in synchronism with the alternating current system, but not in parallel with it, a sudden change of 6 volts in the e.m.f. applied to the armature would mean a tendency to a momentary rush of 1666 amperes through the converter armature. This current, if it should actually flow, would almost instantly change the speed of the converter, and the action would be so rapid that the change in speed would occur before the alternating-current switches could be closed, if it happened that the operation of paralleling was being carried on at this instant. The converter would therefore go in parallel with the alternating-current system when not properly in phase, and whether the machine would pull itself in step or momentarily fall out of step would depend upon its inherent characteristics. In any event there would be a heavy rush of current from both the alternating- and direct-current circuits through the converter armature. The current that would flow from the direct-current circuit would be largely determined by the armature resistance, so that with the low armature resistance in the assumed rotary converter the current would be large.

Assume, in the second place, that when the same rotary converter is brought up to synchronous speed, 1 ohm resistance is left in circuit between the converter armature and the direct-current supply circuit. This would represent the opposite extreme condition. As approximately 40 amperes will be sufficient to maintain the converter at speed under these conditions, this additional resistance will cause only a relatively small drop in voltage. Assuming, as in the first case, a change of 6 volts in the bus bar voltage, the change in current to the converter will be 6 instead of 1666 amperes. With this small current the change in speed due to change in voltage will be entirely due to the consequent change in field strength, and this will occur slowly, so that it will be comparatively easy to synchronize the converter with the alternating-current system even with a fluctuating direct-current voltage. Furthermore, should the converter be paralleled when not exactly in synchronism, while a large current will flow from the alternating current circuit as in the first case, in order to bring the armature into synchronism, only a small current will flow from the direct current side on account of the resistance left in the starting circuit. It should also be noted that with the resistance in the starting circuit it is not necessary to have the voltage of the converter and of the alternating-current circuit the same when they are paralleled. If they are different, it will only require a comparatively small corrective current from the alternating-current circuit to equalize them, and the resulting change in the direct-current voltage of the converter will not cause a disturbance in the direct-current circuit, since the resistance in the starting circuit will limit the current resulting from this difference between the direct-current voltages of the machine and circuit. For this reason it is possible to employ this resistance even when the voltage of the bus bars is the same as, or even less than, the voltage necessary to make the alternating-current voltages of the converter and of the alternating-current system the same.

A rotary converter of low armature resistance in connection with a low-resistance starting circuit will be very sensitive to changes in voltage; the changes in speed will be large and will occur quickly. Synchronizing will be correspondingly difficult. With a starting circuit of relatively large resistance the same converter will be sluggish; changes in voltage will have comparatively little effect on the converter; speed and synchronizing will be comparatively easy. In either case, however, paralleling will not be uniformly successful unless the design of the rotary converter is such that it will pull into step even if the switches are closed when the converter is not exactly in synchronism.

It is evident that with a rotary converter of suitable design and with a starting circuit of suitable resistance, starting from the direct-current side is entirely satisfactory. This is true whether the source of direct current is the service bus bars or a separate starting generator.

The preceding discussion refers particularly to the use of the service bus bars as the source of power for starting. When a separate starting generator is used, the conditions are of the same nature, but are more favorable. There will be little fluctuation in the voltage, and it can be readily adjusted to the necessary value, so that suitable phase relations and voltage conditions can be obtained without difficulty. The starting generator is made shunt wound. This is done in order to obtain a drooping voltage characteristic which gives results very similar to those obtained by a resistance placed between the converter armature and the bus bars. Any increase in load on the starting generator will cause a drop in voltage at its terminals; any decrease in load will cause an instant rise in voltage. Therefore, if there be any tendency to a rush of current between the alternating- and direct-current circuits at the instant of paralleling, the natural voltage characteristics of the starting generator will prevent its occurrence.

With this method of starting by direct current the converter may be permanently connected to the low-tension side of the transformers and paralleling done by means of switches in the high-tension line. When this is done, it will require more current to start the converter than if the transformers were entirely disconnected. The increase in starting current will vary with converters of different numbers of phases. In a two-phase converter the entire armature winding may be paralleled by the transformer coils. The increase of current will be governed by the relative resistance of the two circuits, but in general it will be so great that it is not feasible to start a two-phase converter with the transformers connected. This increase in current exists only at the instant of starting; as the converter begins to revolve alternating current flows in the transformer secondary and the self-induction limits the current to a negligible value. Therefore, the switches in the low-tension transformer circuit may be closed at any time after the converter starts, provided, of course, there are also switches in the high-tension circuit, which is generally the case.

With a three-phase converter only a part of the armature will at any time be paralleled by the transformers, and the current shunted from the armature will be correspondingly less. It is feasible to start with direct current with the transformer secondaries connected, and three-phase converters are often installed in this way. The increase in starting current due to the transformers will depend on the characteristics of individual converters and transformers, but as an approximate rule it may be stated that with three-phase rotary converters the starting current will be increased 100% when the transformer secondaries are connected to the armature. With the transformers disconnected it will require, roughly, 8% of the rated full-load current for starting, and with the transformers connected it will require about 15%. It should be understood that these figures are very approximate. Too much dependence should not be placed upon them, as they will vary considerably in individual cases.

With a six-phase rotary converter and with the secondaries of the transformers connected across diametrical points of the armature winding (Fig. 243) the conditions of starting with the transformers connected to the converter are much the same as with a two-phase converter; it is not considered practicable to start the converter with the transformers so connected. With the transformers connected in the more usual double-delta, the same starting conditions exist as with the delta-connected three-phase converter, and the results will be similar.

(c) *Starting with Separate Alternating-current Motor.*—Where it is essential that the starting current be kept as low as possible because of limited capacity of generators or transmission system, the third method of starting by means of a separate starting motor is sometimes adopted.

A modification of the method (c) is sometimes employed in which the rotary is started from rest by means of a separate starting motor and then thrown directly on to the alternating-current buses in series with a suitable reactance which limits the instantaneous rush of current to a safe value. This method combines the advantages of separate motor starting and self-starting from the alternating-current side, in that it obviates the necessity of synchronizing and insures the rotary coming in with the right polarity; on the other hand, it requires somewhat more time than the self-starting method, and a slightly heavier line current than with the induction starting motor alone.

154. Voltage Regulation of Rotary Converters.—Regulation of the direct-current voltage of a rotary converter can be obtained only by altering the impressed alternating-current potential, owing to the practically constant conversion ratio already mentioned.

Non-automatic regulation can be obtained by means of (*a*) transformer taps, (*b*) potential regulators, (*c*) induction regulators, and (*d*) synchronous regulators or alternating-current boosters. With the necessary auxiliary apparatus the induction and synchronous regulators can be made to operate automatically, while within narrow limits automatic regulation can be obtained by means of (*e*) a properly proportioned series field winding and inductance.

(*a*) *Transformers* for rotary converter service are designed for the normal secondary voltage which will give the required direct-current voltage. They are provided with taps on both primary and secondary windings, which permit compensation for line drop or for any slight variation from the predetermined standard voltage. These taps cannot be used for regulation while the apparatus is in service.

(*b*) The *step type potential regulator* consists of a special transformer having the primary coil connected across the line and the secondary in series with the line. A number of taps are brought out from the secondary to contact points on a face plate, or a drum, and a reversing switch is provided, by means of which the impressed voltage at the rotary slip rings can be raised or lowered without reference to the line voltage. For polyphase circuits three regulators are employed, one in each phase, with the brushes or contact arms mechanically connected so that a single handle will control all phases simultaneously. For automatic voltage control the contact arms may be operated by means of a small motor controlled from the switchboard.

(*c*) The *induction type regulator* is also a special transformer, the primary coils of which are movable with respect to the secondary, so that there is no opening of the circuit between steps. The general construction of core and winding resembles that of an induction motor with the primary coil connected across the line and the secondary in series with the line. By turning the movable primary coil through an angle of 180 electrical degrees, the voltage of the rotary converter will vary from a minimum to a maximum value. (See Westinghouse Bulletin on "induction regulator.")

As the potential at the collector rings and the direct-current voltage vary proportionately, any desirable range within the capacity of the regulator can be obtained without opening the circuits.

For automatic operation or remote control the induction regulator is operated by a small motor. This type of regulator is very satisfactory in overcoming fluctuations of line voltage in lighting and similar service.

(*d*) The *synchronous regulator* consists of an alternating-current generator with the same number of poles as the rotary converter, and having its rotor, or armature, mounted on the rotary converter shaft. The armature windings of this generator, which is of the revolving armature type, are connected in series between the armature and the collector rings of the rotary. The regulator frame with its field poles and windings is supported by brackets attached to the rotary converter frame. This arrangement avoids the use of brushes and collector rings for the regulator and affords a very simple and compact device. (See Westinghouse Bulletin, No. 1028.)

In operation the field of the regulator is separately excited, so that its armature voltage may be varied from zero to a maximum depending on design constants. Thus, since the armature windings of both rotary and regulator are in series, any desired increment may be added to the alternating-current voltage impressed on the rotary armature. By varying the field excitation of the regulator the alternating-current impressed voltage is raised or lowered and the potential of the direct-current system altered accordingly. If necessary, the excitation and polarity of the regulator may be reversed, so that the increment of potential derived from the regulator may be subtracted from the normal value of the impressed alternating-current voltage obtained. The regulator may be arranged for either automatic or manual control.

With a synchronous regulator having 15% of the rotary converter capacity, the direct-current voltage can be varied 30% at will; or, if desired, a constant potential can be maintained. Furthermore, the regulator field can be so controlled that when operating in parallel with other rotaries, or storage batteries, the rotary will carry its constant proportion of the total load. The rotary converter itself is the same as other standard rotary converters, the main field windings being free from all regulating devices. It possesses all the usual advantages with the addition of the greater flexibility of voltage due to the synchronous regulator.

The power factor and wave form of the distributing system are in no way disturbed, nor is the operation of other apparatus interfered with, by the synchronous regulator, hence this method can be adapted to meet the widest range of conditions.

(*e*) *Compounding of Rotary Converters.*—It is a well-known fact that there is a certain adjustment of rotary converter field strength that gives a minimum alternating-current input for a given direct-current output (when E and I are in phase, Fig. 164) and that varying the field strength either way will increase this alternating-current input, while the direct-current output remains the same. In every case the alternating-current input may be considered as being made up of two components: one, the energy component in phase with the impressed electromotive force; the other, a wattless component, leading with respect to the line voltage when the field strength is too great and lagging when the field strength is too small.

A constant alternating-current voltage across the collector rings has been assumed, and it is evident that the constancy of this applied voltage means the generation of practically a constant voltage by the rotation of the rotary converter armature. To generate this constant voltage in the converter armature (which runs at a definite speed) there must be a constant magnetic induction through the armature, and therefore a constant number of ampere-turns to produce that induction.

In a rotary converter, or other synchronous machine, there are two sources of excitation—current in the field winding and current in the armature winding. To produce a constant voltage the sum of these two excitations must be constant. If the ampere-turns in the field winding are less than necessary, a current circulates in the armature in such a manner as to assist the field in magnetizing the armature. Such a magnetizing current, though leading with respect to the electromotive force of the converter regarded as a generator of electromotive force, is lagging with respect to the generator or supply system. On the other hand, if the ampere-turns of the field winding are more than sufficient to produce the constant voltage we have assumed, a current circulates in the armature in such a manner as to oppose the field winding and so reduce the magnetic induction to the required amount. This magnetizing current is leading with respect to the generator or supply system. (See art. 93.)

The actual current circulating in the armature may be regarded as being composed of the magnetizing current and the useful current, and the ratio of this useful component to the resultant of the useful and magnetizing components is known as the power factor of the circuit. It is measured by the ratio of the true output (recorded by a wattmeter) to the apparent input (recorded by an ammeter and voltmeter).

From the above analysis it is evident that the power factor of the alternating current taken by a rotary converter is intimately dependent on the amount of its armature magnetizing or demagnetizing current, and therefore dependent upon the field current of the converter. The limits of variation of power factor are from a very low power factor lagging to a very low power factor leading. The first occurs with a small load or no load when the field excitation is too low, and the second with a small load or no load when the field excitation is too high.

From this it will be seen that variations in shunt field strength give lagging or leading current in the alternating-current line. It does not, however, affect the armature magnetization, and so a change in shunt-field strength alone cannot affect the direct-current voltage. This bears a fixed relation to the alternating-current voltage, so that the only way to raise the direct current voltage to any extent is by raising the alternating-current voltage at the collector rings.

Now, the voltage at the collector rings of a converter is the resultant of two components: first, the voltage of the generator; and second, the voltage needed to force the current taken by the converter through the inductance between itself and the generator. The resistance between the generator and the converter is also a modifying element, but we will temporarily assume this resistance to be negligible, and without attempting at this point to give reasons, state the facts concerning the resultant of the two components.

(1) As long as the current supplied to the converter is neither lagging nor leading, the inductance volts combine with the generator volts in such a manner that their resultant, the converter voltage, is practically the same as the generator voltage; that is, a non-inductive load causes practically no drop over that due to the resistance of the line.

(2) If the converter current is lagging, the two elements combine in such a way that their resultant is smaller than the generator volts; that is, the electromotive force of self-induction causes an increased drop over that due to line resistance.

(3) If the current is leading, the resultant is greater than the generator volts; that is, the e.m.f. of self-induction in this case adds to the generator volts.

It is evident, therefore, that if we can cause the current taken by the rotary converter to change from lagging at no load to leading at full load, the voltage applied to the converter will change from an amount less than the generator voltage at no load to an amount greater than the generator voltage at full load, and this is exactly the function of the series field. It makes the field ampere turns increase with load, and therefore, as set forth above, causes the lagging component of the current supplied to the rotary converter to decrease, or the leading component to increase, or both.

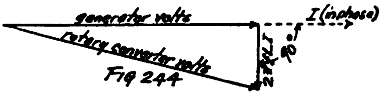

Fig 244

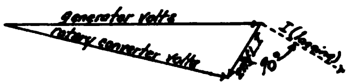

Fig 245

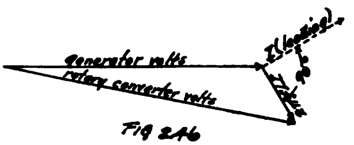

Fig 246

Now, to explain somewhat further the reasons why the resultant of the two components which make up the converter voltage increases as the current changes from lagging to leading. The volts across an inductance (the e.m.f. of self-induction) lag 90° behind the current causing them (the resistance being negligible). This lag remains fixed under all conditions. If, therefore, the current is neither lagging nor leading, with reference to the generator volts, the inductance volts caused by that current lag 90°

behind the generator volts. This condition is shown in Fig. 244, where it is evident that the rotary converter volts are practically of the same value as the generator volts. Now, if the rotary converter current lags, the inductance volts are thrown around into such a position that the resultant of them and the generator volts is smaller than the generator volts. This condition is shown in Fig. 245. If, on the other hand, the rotary converter current is leading, the resultant of the two elements is greater than the generator volts, as shown in Fig. 246.

It will be noted that the amount of compounding depends upon two things:

(1) Upon the variation in power factor between no load and full load, or, referring to the diagrams, upon the change in angle between the generator volts and inductance volts.

(2) Upon the drop across the inductance, or, referring to the diagrams, upon the length of the line representing inductance volts, compared to the length of the line representing generator volts.

The first condition depends upon the relative amount of series winding on the rotary converter. The second, the inductance in circuit depends, however, upon conditions outside of the rotary converter, over some of which there is no control. For this reason it is impossible to make guarantees on the compounding of rotary converters unless conditions governing inductance in the circuit are thoroughly known.

In a rotary converter plant there are usually three sources of inductive drop: the step-up transformers, the line, and the step-down transformers. In specific cases, however, any or all of these sources may be absent. For instance, where a rotary converter is operated directly from a generator in the same building, there will be practically no inductance between the rotary converter and the generator. In order to obtain compounding in this case, it is necessary to insert inductance (usually choke coils) between the rotary converter and the generator.

In any case where this becomes necessary it should be borne in mind that the voltage applied to the rotary converter is not that of the generator alone. The "drop" across the choke coils (inserted for the purpose of giving compounding) should be subtracted from the generator voltage to obtain the true rotary converter volts at no load. The amount to be thus subtracted will depend both upon the amount of inductance in the choke coils and also upon the percentage of full-load current for which the rotary converter shunt field is adjusted at no load.

This reduction of voltage at no load applies to the rotary converter, whether in the same building with the generator, or at the end of a long line. In the latter case, however, the problem is somewhat complicated by the addition of a resistance drop as well as an inductance drop.

The two general statements that follow in regard to the compounding of rotary converters are approximately true.

(1) A rotary converter which will compound 10% when operated as a direct-current generator will require from 12% to 15% inductance in order to make it compound 10% when operated as a converter. This statement assumes that the ohmic drop in the transmission line is negligible. When an appreciable ohmic drop occurs, as, for instance, at the end of a long transmission line, the regulation will be reduced by practically the percentage of the ohmic drop that occurs.

(2) In a rotary converter that will compound 10% as a direct-current generator and is on a circuit with from 12% to 15% inductance, the proper adjustment of the shunt field will be such that at no load the rotary converter takes 30% full-load lagging current. With the shunt field adjustment unchanged, full-load voltage applied, the power factor will be 90% leading.

The question may naturally be raised as to what is meant by 12% to 15% inductance, and how it can be measured. Twelve per cent. inductance in a rotary converter circuit means that it requires 12% of full voltage to force full-load current through the self-induction of the rotary converter circuit. In other words, if the rotary converter be short-circuited at its collector rings, it will require practically 12% of full voltage at the generator to force full-load current of the converter through the circuit (assuming zero resistance). The self-induction of the generator armature is omitted in this latter statement, as in cases where the generator is large compared with the rotary converter (and this is usually the case), the self-induction of the generator armature is an inconsiderable part of the total self-induction.

If at any time it becomes necessary to measure the inductance volts it may be done by short-circuiting at the collector rings and measuring the voltage necessary to force full-load rotary converter current through the circuit. This will include resistance volts as well as inductance volts. To eliminate the resistance element, calculate the percentage of full-load volts that it requires to drive full-load rotary converter current through the circuit to the short-circuited point. The percentage of inductance volts will then be the third leg of a right-angled triangle, of which the percentage of full-load volts measured above is the hypothenuse and the percentage resistance volts is the second leg.

For example, in a 2200-volt system suppose it takes 400 volts at the generator to force the full-load current of a rotary converter through the circuit when the converter is short-circuited at the collector rings. This means 20% of full voltage. Suppose, further, that a calculation shows that it will require 300 volts or 15% of the generator voltage to force the same current through the resistance of the circuit. By making 20 the hypothenuse and 15 one leg of a right-angled triangle we find that about 13.2 is the third leg. That means that there is about 13.2% inductance in that circuit.

The preceding discussion may be briefly recapitulated as follows:

(1) A series coil on a rotary converter has the effect of automatically changing the ampere-turns applied to the field.

(2) Varying the field ampere-turns means varying the magnetizing element of the alternating current delivered to the collector rings, causing the current to become leading when the ampere turns are more than normal and lagging when the ampere turns are less than normal.

(3) The voltage across any inductance in circuit tends to subtract from the generator voltage when the converter current lags, and tends to add to the generator voltage when the converter current leads.

(4) The conditions for rotary converter compounding are therefore:

(*a*) A series winding on the rotary converter connected so as to assist the shunt.

(*b*) Inductance between the generator and rotary converter.

The series field coils in a rotary converter do not increase the voltage directly, as in a direct-current generator, but indirectly, through co-operation with inductance in the supply circuit.

This method of voltage control is automatic within narrow limits.

155. Parallel Operation of Rotary Converters on Direct-current Side.—If several rotary converters are to supply the same direct-current system, they can be connected in parallel in the same manner as shunt or compound direct-current generators. With compound-wound rotaries an equalizing connection, similar to that employed between series fields on direct-current generators, must be made between the different machines. The load will be divided in proportion to the respective voltages, which should be maintained as nearly equal as possible.

It is not advisable to operate rotary converters supplying the same direct-current system in parallel on the alternating-current side; that is, from the secondaries of the same bank of transformers. With such an arrangement a closed local circuit is formed in which heavy cross-currents may be established by a slight change in the operating conditions; as, for example, a slight difference in the relative position of the direct-current brushes on the two machines. Moreover, when two or more rotaries are operated from the same secondary circuit it becomes very difficult, if not impossible, to adjust the alternating-current potential so as to compensate for the slight but inevitable inequality of the direct-current voltages of the several machines.

156. Inverted Rotary Converters.—If desired, rotary converters may be driven by direct current to supply alternating-current circuits. In such inverted operation the speed is not held constant by the alternating-current circuit, but rather follows the general laws of direct-current motors, depending on the strength of the magnetic field and impressed armature voltage. The field strength, in turn, depends upon the field current and the demagnetizing effect of the armature reaction. If the alternating-current load on the rotary is inductive, the armature reactions weaken the field, thus causing an excessive increase in speed and frequency.

Rotaries, when supplied for inverted operation, are provided with a special exciter. This exciter is direct-connected to the rotary shaft or driven by an induction-motor receiving current from the alternating-current side of the rotary; in either case the speed of the exciter is proportional to the speed of the rotary. This exciter is designed for normal operation at a low point on its saturation curve, so that its voltage is considerably increased by any increase in the speed of the rotary. Since the increased exciter voltage increases the strength of the rotary field while the armature voltage remains constant, the rotary tends to slow down. Speed regulation is thus automatic.

157. The Converter Used as a Double-current Generator.—If a converter be supplied with a pulley and driven by a prime mover, it will supply D.C. from its commutator acting as a D.C. generator, and supply A.C. from its collector rings acting as an A.C. generator, and when so used it is called a double-current generator. It is often used in this manner in small testing laboratories where it is desired to obtain D.C. and A.C. without the expense of two machines.

Rotary Converters on a D.C. Three-wire System.—Fig. 247 represents a single-phase converter arranged to operate on a 110-volt three-wire D.C. system. The middle wire of the three-wire system is connected to the middle of the secondary coil of the step-down transformer, which is always at zero potential. With the unbalancing assumed in the figure one ampere of D.C. flows each way from the middle point of the secondary, through the secondary of the transformer, and this D.C. annuls itself in the two halves of the secondary, and therefore does not in any way disturb the magnetization due to the transformer action.

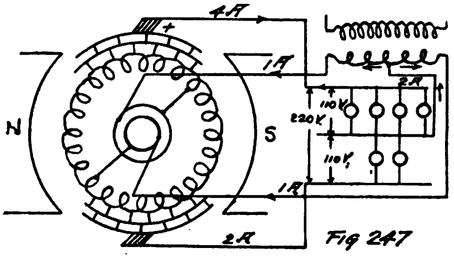

Fig 247

158. Motor Generator Sets.—These consist of an alternating-current motor direct connected to a direct-current generator. The alternating-current motor takes, say, 11,000 volts A.C. and drives the D.C. generator, which delivers current at, say, 600 volts D.C. for railway service.

The A.C. motor may be a polyphase induction motor or it may be a polyphase synchronous motor. If it be an induction motor, the set is, of course, self-starting and need not be synchronized. If it be a polyphase synchronous motor, its field is generally excited direct from the D.C. generator, thus obviating the necessity of an exciter. The fields are generally equipped with an auxiliary squirrel-cage winding, so that the motor will be self-starting at reduced voltage with the same ease as induction motors with squirrel-cage rotors. (See art. 153, *a*.) When so started, they are self-synchronizing.

The chief advantage of the motor generator set over the converter is that the voltage regulation on the D.C. side is entirely independent of the A.C. side. The efficiency is lower and the first cost higher than in the case of a rotary converter.

159. Frequency Changers.—A frequency of twenty-five cycles has been quite generally adopted for railway service; also, in some large cities, sixty-cycle current is generated and distributed to substations in which are installed rotary converters supplying an Edison three-wire network.

Since sixty cycles are more favorable than twenty-five cycles for A.C. lighting, frequency changers are installed to furnish high-tension sixty-cycle current for outlying districts beyond the reach of the three-wire system.

Frequency changers consist of a synchronous motor or an induction motor direct connected to an A.C. generator.

160. The Mercury Arc Rectifier.—The mercury arc rectifier consists of a closed glass vessel provided with four electrodes, the two upper ones (Fig. 248) being of graphite and the two lower ones of mercury. The air is exhausted from the tube and suitable terminals are provided for connecting the electrodes to the circuit. The graphite electrodes are the positive ones; the main mercury electrode is the negative, and the small additional one is called the starting electrode. The bulb is mounted in a cradle so that it can be tilted when it is necessary to start it operating.

The bulb is filled with a highly attenuated vapor of mercury, which is an electrical conductor under some conditions. Current can pass readily from either of the solid electrodes to the vapor, but if an attempt be made to reverse the direction of flow, so that the current tends to flow from the vapor to the solid electrode, there is an apparent resistance at the surface of the electrode which prevents the flow of current.

When the bulb starts to rectify, there is a high resistance at the surface of the mercury known as the negative-electrode resistance, and this acts like a flux over the surface of the mercury. In order to break down this film the bulb is tilted and the space between the two mercury electrodes is bridged; currents then flow, and when the bulb is returned to the vertical position this bridge is broken, producing a spark. This spark breaks down the negative-electrode resistance, after which the rectifier will continue to operate.

At the instant the right-hand side of the secondary is positive, current flows as shown by the full line arrows; one half-cycle later current flows as shown by the dotted arrows. It is seen that the current is unidirectional through the batteries.

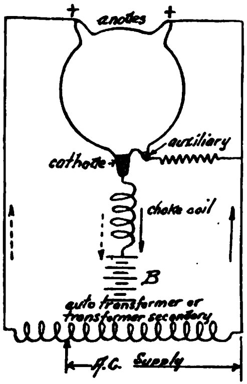

Fig 248

Any interruption to the current, even for the smallest instant of time, permits the negative-electrode resistance to re-establish itself, which stops the operation of the bulb. In order to prevent this, the choke coil is placed as shown, which, of course, tends to keep the current continuous.

A mercury arc rectifier is simpler, cheaper, takes up less space, and is more efficient (70 to 85% efficiency) than a motor generator set (60% efficiency), and under normal conditions the life of the bulb is nearly five hundred hours. (See "Cooper Hewitt Rectifier," Westinghouse bulletin No. 1148.)

PROBLEMS.

188. Make a sketch similar to Fig. 236 for a six-pole (*a*) single-phase, (*b*) two-phase, (*c*) three-phase converter.
189. In Fig. 236, the voltage across each phase is 100 volts. What is the voltage between any A ring and any B ring?
190. Make a sketch showing a two-pole two-phase converter connected to a two-phase three-wire system. What is the difficulty with this arrangement?
191. Derive an expression for the armature current and voltage relations between the A.C. and D.C. sides of an n-phase converter.
192. What is the voltage between ring A and every other ring on the six-phase converter shown in Fig. 243, if the voltage A A′ is 1000 volts?
193. Derive the relation between the A.C. and D.C. armature currents on a six-phase converter. *Ans.*:—$I_{(A.C.)} = .471\, I_{(D.C.)}$.
194. Plot curves similar to Fig. 242, *a* and *b*, for the A.C. and D.C. currents in a coil situated midway between the coil *a* and the coil *b*, Fig. 240. Compare this coil with coil (*a*) and (*b*) as to heating.
195. Make a diagrammatic sketch of a converter operating with an induction type regulator.
196. Make a diagrammatic sketch of a converter operating with a synchronous regulator.
197. Make a diagrammatic sketch of a converter operating with step-down

transformers which are provided with taps on both primary and secondary windings to compensate for line drop.

198. Make a diagrammatic sketch of a converter operating with a step type potential regulator.

199. Make a diagrammatic sketch of a two-phase converter operating on a D.C. three-wire system.

200. Make a diagrammatic sketch for starting a two-phase converter from the D.C. side.

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